第 3 章 3 「連続型確率分布」「連続型確率変数の平均と分散」 第 3 回

解答

1. $a = \dfrac{\sqrt{6}}{3}$ ， $P(0.5 \leqq X \leqq 1.5) = \dfrac{5}{8}$

2. $P(0 \leqq X \leqq 1) = \dfrac{1}{8}$ ， $E[X] = \dfrac{3}{2}$ ， $V[X] = \dfrac{3}{20}$

解説

1.
$$\begin{aligned}\int_{-\infty}^{\infty} f(x)\,dx &= \int_0^a f(x)\,dx \\ &= \int_0^a 3x\,dx \\ &= 3\left[\frac{x^2}{2}\right]_0^a = \frac{3}{2}a^2 = 1 \ (\text{全確率})\end{aligned}$$
$$\therefore \quad a = \frac{\sqrt{6}}{3} \quad (\because a \geqq 0)$$
$$\begin{aligned}P(0.5 \leqq X \leqq 1.5) &= \int_{0.5}^{1.5} f(x)\,dx \\ &= \int_{\frac{1}{2}}^{\frac{\sqrt{6}}{3}} 3x\,dx \\ &= 3\left[\frac{x^2}{2}\right]_{\frac{1}{2}}^{\frac{\sqrt{6}}{3}} \\ &= \frac{3}{2}\left(\frac{6}{9} - \frac{1}{4}\right) = \frac{5}{8}\end{aligned}$$

[注意] $P(0.5 \leqq X \leqq 1.5)$ について，積分範囲が $0.5 \leqq x \leqq \dfrac{\sqrt{6}}{3} (= a)$ となるのは $0.5 < \dfrac{\sqrt{6}}{3} < 1.5$ だからである．この不等式の大小関係は，各辺を 2 乗することで確かめられる．

2. $P(0 \leqq X \leqq 1) = \displaystyle\int_0^1 \frac{3}{8}x^2\,dx = \frac{3}{8}\left[\frac{x^3}{3}\right]_0^1 = \frac{1}{8}$

$$\begin{aligned}E[X] &= \int_{-\infty}^{\infty} xf(x)\,dx \\ &= \int_0^2 x\frac{3}{8}x^2\,dx \\ &= \frac{3}{8}\left[\frac{x^4}{4}\right]_0^2 = \frac{3}{2}\end{aligned}$$
$$\begin{aligned}E[X^2] &= \int_{-\infty}^{\infty} x^2 f(x)\,dx \\ &= \int_0^2 x^2\frac{3}{8}x^2\,dx \\ &= \frac{3}{8}\left[\frac{x^5}{5}\right]_0^2 = \frac{12}{5}\end{aligned}$$
$$V[X] = E[X^2] - \left(E[X]\right)^2 = \frac{12}{5} - \left(\frac{3}{2}\right)^2 = \frac{3}{20}$$