第 5 章 4.「加法定理」 第 2 回

解答

1. (1) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ (2) $\dfrac{\sqrt{2}-\sqrt{6}}{4}$ (3) $-2-\sqrt{3}$

2. (1) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ (2) $-\dfrac{\sqrt{2}+\sqrt{6}}{4}$ (3) $2-\sqrt{3}$

3. (1) $\dfrac{63}{65}$ (2) $\dfrac{56}{65}$ (3) $\dfrac{33}{56}$

解説

1. $\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$, $\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$, $\tan(\alpha\pm\beta)=\dfrac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$

(1) $\sin 105^\circ=\sin(60^\circ+45^\circ)$
$=\sin 60^\circ\cos 45^\circ+\cos 60^\circ\sin 45^\circ$
$=\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}+\dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2}=\dfrac{\sqrt{6}+\sqrt{2}}{4}$

(2) $\cos 105^\circ=\cos(60^\circ+45^\circ)$
$=\cos 60^\circ\cos 45^\circ-\sin 60^\circ\sin 45^\circ$
$=\dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}=\dfrac{\sqrt{2}-\sqrt{6}}{4}$

(3) $\tan 105^\circ=\tan(60^\circ+45^\circ)$
$$=\frac{\tan 60^\circ+\tan 45^\circ}{1-\tan 60^\circ\tan 45^\circ}=\frac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1}=\frac{1+\sqrt{3}}{1-\sqrt{3}}=\frac{(1+\sqrt{3})^2}{(1-\sqrt{3})(1+\sqrt{3})}=\frac{1+2\sqrt{3}+3}{1-3}=\frac{4+2\sqrt{3}}{-2}$$
$=-2-\sqrt{3}$

2. (1) $\sin 75^\circ=\sin(45^\circ+30^\circ)$
$=\sin 45^\circ\cos 30^\circ+\cos 45^\circ\sin 30^\circ$
$=\dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2}+\dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2}=\dfrac{\sqrt{6}+\sqrt{2}}{4}$

(2) $\cos 165^\circ=\cos(120^\circ+45^\circ)$
$=\cos 120^\circ\cos 45^\circ-\sin 120^\circ\sin 45^\circ$
$=\left(-\dfrac{1}{2}\right)\cdot\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}=-\dfrac{\sqrt{2}+\sqrt{6}}{4}$

(3) $\tan 15^\circ=\tan(45^\circ-30^\circ)$
$$=\frac{\tan 45^\circ-\tan 30^\circ}{1+\tan 45^\circ\tan 30^\circ}=\frac{1-\frac{1}{\sqrt{3}}}{1+1\cdot\frac{1}{\sqrt{3}}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}=\frac{3-2\sqrt{3}+1}{3-1}=\frac{4-2\sqrt{3}}{2}$$
$=2-\sqrt{3}$

3. $\cos^2\alpha+\sin^2\alpha=1$ より，$\cos^2\alpha=1-\sin^2\alpha=1-\dfrac{16}{25}=\dfrac{9}{25}$ このとき，α は第 3 象限の角なので，$\cos\alpha<0$

よって $\cos\alpha=-\dfrac{3}{5}$ したがって, $\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}=-\dfrac{4}{5}\div\left(-\dfrac{3}{5}\right)=\dfrac{4}{5}\times\dfrac{5}{3}=\dfrac{4}{3}$

$\cos^2\beta+\sin^2\beta=1$ より，$\sin^2\beta=1-\cos^2\beta=1-\dfrac{144}{169}=\dfrac{25}{169}$ このとき，β は第 2 象限の角なので，$\sin\beta>0$

よって $\sin\beta=\dfrac{5}{13}$ したがって, $\tan\beta=\dfrac{\sin\beta}{\cos\beta}=\dfrac{5}{13}\div\left(-\dfrac{12}{13}\right)=-\dfrac{5}{13}\times\dfrac{13}{12}=-\dfrac{5}{12}$

(1) $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$
$=\left(-\dfrac{4}{5}\right)\cdot\left(-\dfrac{12}{13}\right)-\left(-\dfrac{3}{5}\right)\cdot\dfrac{5}{13}$
$=\dfrac{48}{65}+\dfrac{15}{65}=\dfrac{63}{65}$

(2) $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$
$=\left(-\dfrac{3}{5}\right)\cdot\left(-\dfrac{12}{13}\right)-\left(-\dfrac{4}{5}\right)\cdot\dfrac{5}{13}$
$=\dfrac{36}{65}+\dfrac{20}{65}=\dfrac{56}{65}$

(3) $$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{\frac{4}{3}-\frac{5}{12}}{1-\frac{4}{3}\cdot\left(-\frac{5}{12}\right)}=\frac{\frac{48}{36}-\frac{15}{36}}{1+\frac{20}{36}}=\frac{48-15}{36+20}=\frac{33}{56}$$