# 第３章 8 「極座標による２重積分（発展その２）」 第３回

## 解答

**1.** (1) $2\pi$

(2) $\dfrac{\pi}{14}$

(3) $\dfrac{5}{8}\pi$

## 解説

**1.** (1) $\displaystyle\iint_D x^2 dxdy$

$$
\begin{aligned}
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^2 (r\cos\theta)^2\, r dr d\theta \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left\{ \int_0^2 r^3 dr \right\} \cos^2\theta\, d\theta \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[ \frac{1}{4} r^4 \right]_0^2 \cos^2\theta\, d\theta \\
&= 4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta \\
&= 4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2\theta}{2}\, d\theta \\
&= 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1+\cos 2\theta)\, d\theta \\
&= 2\left[ \theta + \frac{1}{2}\sin 2\theta \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= 2\pi
\end{aligned}
$$

(2) $\displaystyle\iint_D y^2 \left(x^2+y^2\right) \sqrt{x^2+y^2} dxdy$

$$
\begin{aligned}
&= \int_0^{\pi} \int_0^1 (r\sin\theta)^2\, r^2 \sqrt{r^2}\, r dr d\theta \\
&= \int_0^{\pi} \left\{ \int_0^1 r^6 dr \right\} \sin^2\theta\, d\theta \\
&= \int_0^{\pi} \left[ \frac{1}{7} r^7 \right]_0^1 \sin^2\theta\, d\theta \\
&= \frac{1}{7}\int_0^{\pi} \sin^2\theta\, d\theta \\
&= \frac{1}{7}\int_0^{\pi} \frac{1-\cos 2\theta}{2}\, d\theta \\
&= \frac{1}{14}\int_0^{\pi} (1-\cos 2\theta)\, d\theta \\
&= \frac{1}{14}\left[ \theta - \frac{1}{2}\sin 2\theta \right]_0^{\pi} \\
&= \frac{\pi}{14}
\end{aligned}
$$

(3) $\displaystyle\iint_D \frac{y^2}{x^2+y^2} dxdy$

$$
\begin{aligned}
&= \int_0^{\frac{\pi}{2}} \int_2^3 \frac{(r\sin\theta)^2}{r^2}\, r dr d\theta \\
&= \int_0^{\frac{\pi}{2}} \left\{ \int_2^3 r dr \right\} \sin^2\theta\, d\theta \\
&= \int_0^{\frac{\pi}{2}} \left[ \frac{1}{2} r^2 \right]_2^3 \sin^2\theta\, d\theta \\
&= \frac{5}{2}\int_0^{\frac{\pi}{2}} \sin^2\theta\, d\theta \\
&= \frac{5}{2}\int_0^{\frac{\pi}{2}} \frac{1-\cos 2\theta}{2}\, d\theta \\
&= \frac{5}{4}\int_0^{\frac{\pi}{2}} (1-\cos 2\theta)\, d\theta \\
&= \frac{5}{4}\left[ \theta - \frac{1}{2}\sin 2\theta \right]_0^{\frac{\pi}{2}} \\
&= \frac{5}{8}\pi
\end{aligned}
$$