# 第3章 8 「極座標による2重積分（発展その2）」 第1回

## 解答

**1.** (1) $\dfrac{\pi}{16}$

(2) $\dfrac{\pi}{4}$

(3) $\dfrac{\pi}{24}$

## 解説

**1.** (1) $\displaystyle\iint_D y^2 dxdy$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 (r\sin\theta)^2 \, rdrd\theta$$
$$= \int_0^{\frac{\pi}{2}} \left\{ \int_0^1 r^3 dr \right\} \sin^2\theta \, d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left[ \frac{1}{4} r^4 \right]_0^1 \sin^2\theta \, d\theta$$
$$= \frac{1}{4} \int_0^{\frac{\pi}{2}} \sin^2\theta \, d\theta$$
$$= \frac{1}{4} \int_0^{\frac{\pi}{2}} \frac{1-\cos 2\theta}{2} \, d\theta$$
$$= \frac{1}{8} \int_0^{\frac{\pi}{2}} (1-\cos 2\theta) \, d\theta$$
$$= \frac{1}{8} \left[ \theta - \frac{1}{2}\sin 2\theta \right]_0^{\frac{\pi}{2}}$$
$$= \frac{\pi}{16}$$

(2) $\displaystyle\iint_D x^2 dxdy$

$$= \int_0^{\frac{\pi}{2}} \int_0^{\sqrt{2}} (r\cos\theta)^2 \, rdrd\theta$$
$$= \int_0^{\frac{\pi}{2}} \left\{ \int_0^{\sqrt{2}} r^3 dr \right\} \cos^2\theta \, d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left[ \frac{1}{4} r^4 \right]_0^{\sqrt{2}} \cos^2\theta \, d\theta$$
$$= \int_0^{\frac{\pi}{2}} \cos^2\theta \, d\theta$$
$$= \int_0^{\frac{\pi}{2}} \frac{1+\cos 2\theta}{2} \, d\theta$$
$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} (1+\cos 2\theta) \, d\theta$$
$$= \frac{1}{2} \left[ \theta + \frac{1}{2}\sin 2\theta \right]_0^{\frac{\pi}{2}}$$
$$= \frac{\pi}{4}$$

(3) $\displaystyle\iint_D y^2 \left(x^2+y^2\right) dxdy$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 (r\sin\theta)^2 \, r^2 rdrd\theta$$
$$= \int_0^{\frac{\pi}{2}} \left\{ \int_0^1 r^5 dr \right\} \sin^2\theta \, d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left[ \frac{1}{6} r^6 \right]_0^1 \sin^2\theta \, d\theta$$
$$= \frac{1}{6} \int_0^{\frac{\pi}{2}} \sin^2\theta \, d\theta$$
$$= \frac{1}{6} \int_0^{\frac{\pi}{2}} \frac{1-\cos 2\theta}{2} \, d\theta$$
$$= \frac{1}{12} \int_0^{\frac{\pi}{2}} (1-\cos 2\theta) \, d\theta$$
$$= \frac{1}{12} \left[ \theta - \frac{1}{2}\sin 2\theta \right]_0^{\frac{\pi}{2}}$$
$$= \frac{\pi}{24}$$