第3章 7 「極座標による2重積分（発展その1）」 第1回

解答

1. (1) $\dfrac{1}{24}$

(2) 2

(3) $\dfrac{1}{15}$

解説

1. (1) $\displaystyle\iint_D x^3 y\,dxdy$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 (r\cos\theta)^3\ r\sin\theta\, r dr d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left\{\int_0^1 r^5 dr\right\} \cos^3\theta\ \sin\theta\, d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left[\frac{1}{6} r^6\right]_0^1 \cos^3\theta\ \sin\theta\, d\theta$$
$$= \frac{1}{6}\int_0^{\frac{\pi}{2}} \cos^3\theta\ \sin\theta\, d\theta$$
$$= \frac{1}{6}\left[-\frac{1}{4}\cos^4\theta\right]_0^{\frac{\pi}{2}}$$
$$= \frac{1}{24}$$

(2) $\displaystyle\iint_D xy\,dxdy$

$$= \int_0^{\frac{\pi}{2}} \int_0^2 r\cos\theta\, r\sin\theta\, r dr d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left\{\int_0^2 r^3 dr\right\} \cos\theta\ \sin\theta\, d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left[\frac{1}{4} r^4\right]_0^2 \cos\theta\ \sin\theta\, d\theta$$
$$= 4\int_0^{\frac{\pi}{2}} \cos\theta\ \sin\theta\, d\theta$$
$$= 4\left[\frac{1}{2}\sin^2\theta\right]_0^{\frac{\pi}{2}}$$
$$= 2$$

(3) $\displaystyle\iint_D xy^2\,dxdy$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 r\cos\theta\ (r\sin\theta)^2\ r dr d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left\{\int_0^1 r^4 dr\right\} \cos\theta\ \sin^2\theta\, d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left[\frac{1}{5} r^5\right]_0^1 \cos\theta\ \sin^2\theta\, d\theta$$
$$= \frac{1}{5}\int_0^{\frac{\pi}{2}} \cos\theta\ \sin^2\theta\, d\theta$$
$$= \frac{1}{5}\left[\frac{1}{3}\sin^3\theta\right]_0^{\frac{\pi}{2}}$$
$$= \frac{1}{15}$$

注意 (1) ～ (3) の θ の積分はそれぞれ

(1) $\cos\theta = t$ とおいて置換積分，または

$$\frac{d}{d\theta}\left[-\frac{1}{4}\cos^4\theta\right]$$
$$= -\frac{1}{4}\cdot 4\cos^3\theta\frac{d}{d\theta}\cos\theta = \cos^3\theta\ \sin\theta$$

(2) $\sin\theta = t$ とおいて置換積分，または

$$\frac{d}{d\theta}\left[\frac{1}{2}\sin^2\theta\right]$$
$$= \frac{1}{2}\cdot 2\sin\theta\frac{d}{d\theta}\sin\theta = \sin\theta\ \cos\theta$$
$$= \cos\theta\ \sin\theta$$

(3) $\sin\theta = t$ とおいて置換積分，または

$$\frac{d}{d\theta}\left[\frac{1}{3}\sin^3\theta\right]$$
$$= \frac{1}{3}\cdot 3\sin^2\theta\frac{d}{d\theta}\sin\theta = \sin^2\theta\cos\theta$$

の関係を用いている．