第3章 6 「極座標による2重積分（その3）」 第2回

解答

1. (1) $\frac{125}{3}$

(2) 2

(3) $\frac{5}{2}$

(4) $\frac{2}{9}$

(5) $\frac{1}{2}$

解説

1. (1) $\iint_D x dxdy$

$$= \int_0^{\frac{\pi}{2}} \int_0^5 r\cos\theta \, r dr d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left\{ \int_0^5 r^2 \cos\theta dr \right\} d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left\{ \int_0^5 r^2 dr \right\} \cos\theta \, d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left[\frac{1}{3} r^3 \right]_0^5 \cos\theta \, d\theta$$
$$= \frac{125}{3} \int_0^{\frac{\pi}{2}} \cos\theta d\theta$$
$$= \frac{125}{3} \left[\ \sin\theta \ \right]_0^{\frac{\pi}{2}}$$
$$= \frac{125}{3}(1-0)$$
$$= \frac{125}{3}$$

(2) $\iint_D \frac{y}{x^2+y^2} dxdy$

$$= \int_0^{\pi} \int_3^4 \frac{r\sin\theta}{r^2} r dr d\theta$$
$$= \int_0^{\pi} \left\{ \int_3^4 \sin\theta dr \right\} d\theta$$
$$= \int_0^{\pi} \left\{ \int_3^4 dr \right\} \sin\theta \, d\theta$$
$$= \int_0^{\pi} \left[\ r \ \right]_3^4 \sin\theta \, d\theta$$
$$= \int_0^{\pi} \sin\theta \, d\theta$$
$$= \left[-\cos\theta \right]_0^{\pi}$$
$$= -(-1) - (-1)$$
$$= 2$$

(3) $\iint_D y\sqrt{x^2+y^2} dxdy$

$$= \int_0^{\pi} \int_{\sqrt{2}}^{\sqrt{3}} r\sin\theta \sqrt{r^2} \, r dr d\theta$$
$$= \int_0^{\pi} \left\{ \int_{\sqrt{2}}^{\sqrt{3}} r^3 \sin\theta dr \right\} d\theta$$
$$= \int_0^{\pi} \left\{ \int_{\sqrt{2}}^{\sqrt{3}} r^3 dr \right\} \sin\theta \, d\theta$$
$$= \int_0^{\pi} \left[\frac{1}{4} r^4 \right]_{\sqrt{2}}^{\sqrt{3}} \sin\theta \, d\theta$$
$$= \frac{5}{4} \int_0^{\pi} \sin\theta \, d\theta$$
$$= \frac{5}{4} \left[-\cos\theta \right]_0^{\pi}$$
$$= \frac{5}{4} \{-(-1) - (-1)\}$$
$$= \frac{5}{2}$$

(4) $\iint_D x\left(x^2+y^2\right)^3 dxdy$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^1 r\cos\theta \, (r^2)^3 r dr d\theta$$
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left\{ \int_0^1 r^8 \cos\theta dr \right\} d\theta$$
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left\{ \int_0^1 r^8 dr \right\} \cos\theta \, d\theta$$
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\frac{1}{9} r^9 \right]_0^1 \cos\theta \, d\theta$$
$$= \frac{1}{9} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta d\theta$$
$$= \frac{1}{9} \left[\ \sin\theta \ \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$
$$= \frac{1}{9} \{1 - (-1)\}$$
$$= \frac{2}{9}$$

(5) $\iint_D \frac{y}{\left(x^2+y^2\right)^2} dxdy$

$$= \int_0^{\frac{\pi}{2}} \int_1^2 \frac{r\sin\theta}{(r^2)^2} r dr d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left\{ \int_1^2 \frac{1}{r^2} \sin\theta dr \right\} d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left\{ \int_1^2 \frac{1}{r^2} dr \right\} \sin\theta \, d\theta$$
$$= \int_0^{\frac{\pi}{2}} \left[-\frac{1}{r} \right]_1^2 \sin\theta \, d\theta$$
$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} \sin\theta \, d\theta$$
$$= \frac{1}{2} \left[-\cos\theta \right]_0^{\frac{\pi}{2}}$$
$$= \frac{1}{2} \{-0 - (-1)\}$$
$$= \frac{1}{2}$$