第３章 ６ 「極座標による２重積分（その３）」 第１回

解答

1. (1) 72

(2) 2

(3) $\frac{3}{2}$

(4) $\frac{2}{7}$

解説

1. (1) $\iint_D ydxdy$

$$= \int_0^{\frac{\pi}{2}}\int_0^6 r\sin\theta\, rdrd\theta$$
$$= \int_0^{\frac{\pi}{2}}\left\{\int_0^6 r^2\sin\theta dr\right\}d\theta$$
$$= \int_0^{\frac{\pi}{2}}\left\{\int_0^6 r^2 dr\right\}\sin\theta\, d\theta$$
$$= \int_0^{\frac{\pi}{2}}\left[\frac{1}{3}r^3\right]_0^6\sin\theta\, d\theta$$
$$= 72\int_0^{\frac{\pi}{2}}\sin\theta\, d\theta$$
$$= 72\Big[-\cos\theta\Big]_0^{\frac{\pi}{2}} = 72\{-0-(-1)\}$$
$$= 72$$

(2) $\iint_D \frac{x}{x^2+y^2}dxdy$

$$= \int_0^{\frac{\pi}{2}}\int_1^3 \frac{r\cos\theta}{r^2}\, rdrd\theta$$
$$= \int_0^{\frac{\pi}{2}}\left\{\int_1^3 \cos\theta dr\right\}d\theta$$
$$= \int_0^{\frac{\pi}{2}}\left\{\int_1^3 dr\right\}\cos\theta\, d\theta$$
$$= \int_0^{\frac{\pi}{2}}\Big[\ r\ \Big]_1^3\cos\theta\, d\theta$$
$$= 2\int_0^{\frac{\pi}{2}}\cos\theta\, d\theta$$
$$= 2\Big[\sin\theta\Big]_0^{\frac{\pi}{2}} = 2(1-0)$$
$$= 2$$

(3) $\iint_D x\sqrt{x^2+y^2}dxdy$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_1^{\sqrt{2}} r\cos\theta\sqrt{r^2}\, rdrd\theta$$
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left\{\int_1^{\sqrt{2}} r^3\cos\theta dr\right\}d\theta$$
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left\{\int_1^{\sqrt{2}} r^3 dr\right\}\cos\theta\, d\theta$$
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[\frac{1}{4}r^4\right]_1^{\sqrt{2}}\cos\theta\, d\theta$$
$$= \frac{3}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\theta\, d\theta$$
$$= \frac{3}{4}\Big[\sin\theta\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$
$$= \frac{3}{4}\{1-(-1)\}$$
$$= \frac{3}{2}$$

(4) $\iint_D y\left(x^2+y^2\right)^2dxdy$

$$= \int_0^{\pi}\int_0^1 r\sin\theta\,(r^2)^2 rdrd\theta$$
$$= \int_0^{\pi}\left\{\int_0^1 r^6\sin\theta dr\right\}d\theta$$
$$= \int_0^{\pi}\left\{\int_0^1 r^6 dr\right\}\sin\theta\, d\theta$$
$$= \int_0^{\pi}\left[\frac{1}{7}r^7\right]_0^1\sin\theta\, d\theta$$
$$= \frac{1}{7}\int_0^{\pi}\sin\theta d\theta$$
$$= \frac{1}{7}\Big[\ -\cos\theta\ \Big]_0^{\pi}$$
$$= \frac{1}{7}\{-(-1)-(-1)\}$$
$$= \frac{2}{7}$$