第3章 5 「極座標による2重積分（その2)」 第3回

解答

1. (1) 28π

(2) $\dfrac{\pi}{20}$

(3) 2π

(4) $\dfrac{3}{32}\pi$

(5) $\dfrac{\pi}{4}\log 2$

解説

1. (1)
$$\begin{aligned}
&\iint_D dxdy \\
&= \int_0^{2\pi}\int_6^8 r dr d\theta \\
&= \int_0^{2\pi}\left\{\int_6^8 r dr\right\} d\theta \\
&= \int_0^{2\pi}\left[\frac{1}{2}r^2\right]_6^8 d\theta \\
&= 14\int_0^{2\pi} d\theta \\
&= 14\Big[\ \theta\ \Big]_0^{2\pi} \\
&= 28\pi
\end{aligned}$$

(2)
$$\begin{aligned}
&\iint_D \left(x^2+y^2\right)^4 dxdy \\
&= \int_0^{\frac{\pi}{2}}\int_0^1 (r^2)^4 r dr d\theta \\
&= \int_0^{\frac{\pi}{2}}\left\{\int_0^1 r^9 dr\right\} d\theta \\
&= \int_0^{\frac{\pi}{2}}\left[\frac{1}{10}r^{10}\right]_0^1 d\theta \\
&= \frac{1}{10}\int_0^{\frac{\pi}{2}} d\theta \\
&= \frac{1}{10}\Big[\ \theta\ \Big]_0^{\frac{\pi}{2}} \\
&= \frac{\pi}{20}
\end{aligned}$$

(3)
$$\begin{aligned}
&\iint_D \frac{1}{\sqrt{x^2+y^2}} dxdy \\
&= \int_0^{\pi}\int_2^4 \frac{1}{\sqrt{r^2}} r dr d\theta \\
&= \int_0^{\pi}\left\{\int_2^4 dr\right\} d\theta \\
&= \int_0^{\pi}\Big[\ r\ \Big]_2^4 d\theta \\
&= 2\int_0^{\pi} d\theta \\
&= 2\Big[\ \theta\ \Big]_0^{\pi} \\
&= 2\pi
\end{aligned}$$

(4)
$$\begin{aligned}
&\iint_D \frac{1}{\left(x^2+y^2\right)^3} dxdy \\
&= \int_0^{\frac{\pi}{2}}\int_1^{\sqrt{2}} \frac{1}{(r^2)^3} r dr d\theta \\
&= \int_0^{\frac{\pi}{2}}\left\{\int_1^{\sqrt{2}} \frac{1}{r^5} dr\right\} d\theta \\
&= \int_0^{\frac{\pi}{2}}\left[-\frac{1}{4}\frac{1}{r^4}\right]_1^{\sqrt{2}} d\theta \\
&= \int_0^{\frac{\pi}{2}}\left(-\frac{1}{16}+\frac{1}{4}\right) d\theta \\
&= \frac{3}{16}\int_0^{\frac{\pi}{2}} d\theta \\
&= \frac{3}{16}\Big[\ \theta\ \Big]_0^{\frac{\pi}{2}} \\
&= \frac{3}{32}\pi
\end{aligned}$$

(5)
$$\begin{aligned}
&\iint_D \frac{1}{x^2+y^2} dxdy \\
&= \int_0^{\frac{\pi}{2}}\int_{\sqrt{2}}^2 \frac{1}{r^2} r dr d\theta \\
&= \int_0^{\frac{\pi}{2}}\left\{\int_{\sqrt{2}}^2 \frac{1}{r} dr\right\} d\theta \\
&= \int_0^{\frac{\pi}{2}}\Big[\log|r|\Big]_{\sqrt{2}}^2 d\theta \\
&= \int_0^{\frac{\pi}{2}}\left(\log 2-\log\sqrt{2}\right) d\theta \\
&= \log\frac{2}{\sqrt{2}}\int_0^{\frac{\pi}{2}} d\theta \\
&= \log\sqrt{2}\Big[\ \theta\ \Big]_0^{\frac{\pi}{2}} \\
&= \frac{\pi}{2}\log\sqrt{2} = \frac{\pi}{2}\log 2^{\frac{1}{2}} = \frac{\pi}{2}\cdot\frac{1}{2}\cdot\log 2 \\
&= \frac{\pi}{4}\log 2
\end{aligned}$$