第3章 3 「2重積分の計算（その3)」 第1回

解答

1. (1) $\dfrac{4}{3}$

(2) $\dfrac{1}{4}$

(3) $\dfrac{1}{12}$

(4) $\dfrac{\pi}{2}$

解説

1. (1)
$$\begin{aligned}
&\iint_D y\,dxdy \\
&= \int_0^2 \left\{ \int_0^x y dy \right\} dx \\
&= \int_0^2 \left[\frac{1}{2} y^2 \right]_0^x dx \\
&= \int_0^2 \frac{1}{2} x^2 dx \\
&= \left[\frac{1}{6} x^3 \right]_0^2 \\
&= \frac{4}{3}
\end{aligned}$$

(2)
$$\begin{aligned}
&\iint_D y\,dxdy \\
&= \int_0^1 \left\{ \int_0^{y^2} y dx \right\} dy \\
&= \int_0^1 \left[yx \right]_0^{y^2} dy \\
&= \int_0^1 y^3 dy \\
&= \left[\frac{1}{4} y^4 \right]_0^1 \\
&= \frac{1}{4}
\end{aligned}$$

(3)
$$\begin{aligned}
&\iint_D x\,dxdy \\
&= \int_0^1 \left\{ \int_{x^2}^x x dy \right\} dx \\
&= \int_0^1 \left[xy \right]_{x^2}^x dx \\
&= \int_0^1 \left(x^2 - x^3 \right) dx \\
&= \left[\frac{1}{3} x^3 - \frac{1}{4} x^4 \right]_0^1 \\
&= \frac{1}{12}
\end{aligned}$$

(4)
$$\begin{aligned}
&\iint_D x\,dxdy \\
&= \int_0^{\frac{\pi}{2}} \left\{ \int_0^{2\cos y} x dx \right\} dy \\
&= \int_0^{\frac{\pi}{2}} \left[\frac{1}{2} x^2 \right]_0^{2\cos y} dy \\
&= \int_0^{\frac{\pi}{2}} 2\cos^2 y\,dy \\
&= \int_0^{\frac{\pi}{2}} 2 \cdot \frac{1+\cos 2y}{2} dy \\
&= \int_0^{\frac{\pi}{2}} (1+\cos 2y)\,dy \\
&= \left[y + \frac{1}{2} \sin 2y \right]_0^{\frac{\pi}{2}} \\
&= \left(\frac{\pi}{2} + \frac{1}{2} \sin \pi \right) - \left(0 + \frac{1}{2} \sin 0 \right) \\
&= \frac{\pi}{2}
\end{aligned}$$