第3章 2 「2重積分の計算（その2）」 第3回

解答

1. (1) -2

(2) 0

(3) $\frac{1}{2}\left(e^3-e^2-e+1\right)$

(4) $e-1$

解説

1. (1)
$$\begin{aligned}
&\iint_D \cos(x+y)dxdy\\
&=\int_0^{\frac{\pi}{2}}\left\{\int_0^{\pi}\cos(x+y)dy\right\}dx\\
&=\int_0^{\frac{\pi}{2}}\Big[\sin(x+y)\Big]_0^{\pi}dx\\
&=\int_0^{\frac{\pi}{2}}\Big\{\sin(x+\pi)-\sin x\Big\}dx\\
&=\Big[-\cos(x+\pi)+\cos x\Big]_0^{\frac{\pi}{2}}\\
&=\left(-\cos\frac{3}{2}\pi+\cos\frac{\pi}{2}\right)-(-\cos\pi+\cos 0)\\
&=(-0+0)-\{-(-1)+1\}\\
&=-2
\end{aligned}$$

(2)
$$\begin{aligned}
&\iint_D \sin(x-y)dxdy\\
&=\int_0^{\frac{\pi}{2}}\left\{\int_0^{\frac{\pi}{2}}\sin(x-y)dy\right\}dx\\
&=\int_0^{\frac{\pi}{2}}\Big[\cos(x-y)\Big]_0^{\frac{\pi}{2}}dx\\
&=\int_0^{\frac{\pi}{2}}\left\{\cos\left(x-\frac{\pi}{2}\right)-\cos x\right\}dx\\
&=\left[\sin\left(x-\frac{\pi}{2}\right)-\sin x\right]_0^{\frac{\pi}{2}}\\
&=\left(\sin 0-\sin\frac{\pi}{2}\right)-\left\{\sin\left(-\frac{\pi}{2}\right)-\sin 0\right\}\\
&=(0-1)-(-1-0)\\
&=0
\end{aligned}$$

(3)
$$\begin{aligned}
&\iint_D e^{x+2y}dxdy\\
&=\int_0^1\left\{\int_0^1 e^{x+2y}dy\right\}dx\\
&=\int_0^1\left[\frac{1}{2}e^{x+2y}\right]_0^1dx\\
&=\frac{1}{2}\int_0^1\left(e^{x+2}-e^x\right)dx\\
&=\frac{1}{2}\left[e^{x+2}-e^x\right]_0^1\\
&=\frac{1}{2}\left\{\left(e^3-e\right)-\left(e^2-1\right)\right\}\\
&=\frac{1}{2}\left(e^3-e^2-e+1\right)
\end{aligned}$$

(4)
$$\begin{aligned}
&\iint_D e^x\cos y\,dxdy\\
&=\int_0^1\left\{\int_0^{\frac{\pi}{2}}e^x\cos y\,dy\right\}dx\\
&=\int_0^1\Big[e^x\sin y\Big]_0^{\frac{\pi}{2}}dx\\
&=\int_0^1\left(e^x\sin\frac{\pi}{2}-e^x\sin 0\right)dx\\
&=\int_0^1 e^x dx\\
&=\Big[e^x\Big]_0^1\\
&=e-1
\end{aligned}$$