第3章 2 「2重積分の計算（その2）」 第2回

解答

1. (1) 2

(2) 0

(3) $e^3 - e^2 - e + 1$

(4) $e - 2$

解説

1. (1) $\displaystyle\iint_D \sin(x+y)dxdy$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left\{ \int_0^{\frac{\pi}{2}} \sin(x+y)dy \right\} dx$$
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Big[-\cos(x+y) \Big]_0^{\frac{\pi}{2}} dx$$
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left\{ -\cos\left(x+\frac{\pi}{2}\right) + \cos x \right\} dx$$
$$= \left[-\sin\left(x+\frac{\pi}{2}\right) + \sin x \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$
$$= \left(-\sin\pi + \sin\frac{\pi}{2} \right) - \left(-\sin 0 + \sin\left(-\frac{\pi}{2}\right) \right)$$
$$= (-0+1) - \{-0+(-1)\}$$
$$= 2$$

(2) $\displaystyle\iint_D \cos(x+2y)dxdy$

$$= \int_0^{\frac{\pi}{2}} \left\{ \int_0^{\pi} \cos(x+2y)dy \right\} dx$$
$$= \int_0^{\frac{\pi}{2}} \left[\frac{1}{2}\sin(x+2y) \right]_0^{\pi} dx$$
$$= \int_0^{\frac{\pi}{2}} \left\{ \frac{1}{2}\sin(x+2\pi) - \frac{1}{2}\sin x \right\} dx$$
$$= \left[-\frac{1}{2}\cos(x+2\pi) + \frac{1}{2}\cos x \right]_0^{\frac{\pi}{2}}$$
$$= \left(-\frac{1}{2}\cos\frac{5}{2}\pi + \frac{1}{2}\cos\frac{\pi}{2} \right) - \left(-\frac{1}{2}\cos 2\pi + \frac{1}{2}\cos 0 \right)$$
$$= \left(-\frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 0 \right) - \left(-\frac{1}{2}\cdot 1 + \frac{1}{2}\cdot 1 \right)$$
$$= 0$$

(3) $\displaystyle\iint_D e^{x+y}dxdy$

$$= \int_0^2 \left\{ \int_0^1 e^{x+y}dy \right\} dx$$
$$= \int_0^2 \Big[e^{x+y} \Big]_0^1 dx$$
$$= \int_0^2 \left(e^{x+1} - e^x \right) dx$$
$$= \Big[e^{x+1} - e^x \Big]_0^2$$
$$= \left(e^3 - e^2\right) - (e-1)$$
$$= e^3 - e^2 - e + 1$$

(4) $\displaystyle\iint_D xe^{xy}dxdy$

$$= \int_0^1 \left\{ \int_0^1 xe^{xy}dy \right\} dx$$
$$= \int_0^1 \Big[e^{xy} \Big]_0^1 dx$$
$$= \int_0^1 \left(e^x - 1 \right) dx$$
$$= \Big[e^x - x \Big]_0^1$$
$$= (e-1) - (1-0)$$
$$= e - 2$$