第 3 章 2 「2 重積分の計算（その 2）」 第 1 回

解答

1. (1) 0

(2) 1

(3) $\frac{1}{2}\left(e^3 - e^2 - e + 1\right)$

解説

1. (1) $\iint_D \cos(x+y)dxdy$

$$= \int_0^{\frac{\pi}{2}} \left\{ \int_0^{\frac{\pi}{2}} \cos(x+y)dy \right\} dx$$
$$= \int_0^{\frac{\pi}{2}} \Big[\sin(x+y) \Big]_0^{\frac{\pi}{2}} dx$$
$$= \int_0^{\frac{\pi}{2}} \left\{ \sin\left(x + \frac{\pi}{2}\right) - \sin x \right\} dx$$
$$= \left[-\cos\left(x + \frac{\pi}{2}\right) + \cos x \right]_0^{\frac{\pi}{2}}$$
$$= \left(-\cos\pi + \cos\frac{\pi}{2} \right) - \left(-\cos\frac{\pi}{2} + \cos 0 \right)$$
$$= \{-(-1) + 0\} - (-0 + 1)$$
$$= 0$$

(2) $\iint_D \sin(2x+y)dxdy$

$$= \int_0^{\frac{\pi}{2}} \left\{ \int_0^{\frac{\pi}{2}} \sin(2x+y)dy \right\} dx$$
$$= \int_0^{\frac{\pi}{2}} \Big[-\cos(2x+y) \Big]_0^{\frac{\pi}{2}} dx$$
$$= \int_0^{\frac{\pi}{2}} \left\{ -\cos\left(2x + \frac{\pi}{2}\right) + \cos 2x \right\} dx$$
$$= \left[-\frac{1}{2}\sin\left(2x + \frac{\pi}{2}\right) + \frac{1}{2}\sin 2x \right]_0^{\frac{\pi}{2}}$$
$$= \left(-\frac{1}{2}\sin\frac{3}{2}\pi + \frac{1}{2}\sin\pi \right) - \left(-\frac{1}{2}\sin\frac{\pi}{2} + \frac{1}{2}\sin 0 \right)$$
$$= \left(-\frac{1}{2}\cdot(-1) + \frac{1}{2}\cdot 0 \right) - \left(-\frac{1}{2}\cdot 1 + \frac{1}{2}\cdot 0 \right)$$
$$= 1$$

(3) $\iint_D e^{2x+y}dxdy$

$$= \int_0^1 \left\{ \int_0^1 e^{2x+y}dy \right\} dx$$
$$= \int_0^1 \Big[e^{2x+y} \Big]_0^1 dx$$
$$= \int_0^1 \left(e^{2x+1} - e^{2x} \right) dx$$
$$= \left[\frac{1}{2}e^{2x+1} - \frac{1}{2}e^{2x} \right]_0^1$$
$$= \left(\frac{1}{2}e^3 - \frac{1}{2}e^2 \right) - \left(\frac{1}{2}e - \frac{1}{2} \right)$$
$$= \frac{1}{2}\left(e^3 - e^2 - e + 1 \right)$$