第2章 3 「高次偏導関数」 第1回

解答

1. (1) $z_{xx} = 12x^2 - 4y, z_{xy} = -4x + 3y^2$

(2) $z_{xx} = -\dfrac{1}{\sqrt{(2x-y)^3}}, z_{xy} = \dfrac{1}{2\sqrt{(2x-y)^3}}$

(3) $z_{xx} = -\dfrac{1}{(x+y)^2}, z_{xy} = -\dfrac{1}{(x+y)^2}$

(4) $z_{xx} = 9e^{3x}\cos 2y, z_{xy} = -6e^{3x}\sin 2y$

2. (1) $z_{xx} = 6, z_{xy} = 4, z_{yx} = 4, z_{yy} = -4$

(2) $z_{xx} = -\sin x \sin y, z_{xy} = \cos x \cos y,$

$z_{yx} = \cos x \cos y, z_{yy} = -\sin x \sin y$

解説

1. (1) $z_x = (x^4 - 2x^2y + xy^3)_x = 4x^3 - 4xy + y^3$

$z_{xx} = (4x^3 - 4xy + y^3)_x = 12x^2 - 4y$

$z_{xy} = (4x^3 - 4xy + y^3)_y = -4x + 3y^2$

(2) $z_x = \{(2x-y)^{\frac{1}{2}}\}_x = \dfrac{1}{2}(2x-y)^{-\frac{1}{2}}(2x-y)_x$

$= \dfrac{2}{2\sqrt{2x-y}} = \dfrac{1}{\sqrt{2x-y}}$

$z_{xx} = \{(2x-y)^{-\frac{1}{2}}\}_x$

$= -\dfrac{1}{2}(2x-y)^{-\frac{3}{2}}(2x-y)_x = -\dfrac{1}{\sqrt{(2x-y)^3}}$

$z_{xy} = \{(2x-y)^{-\frac{1}{2}}\}_y$

$= -\dfrac{1}{2}(2x-y)^{-\frac{3}{2}}(2x-y)_y = \dfrac{1}{2\sqrt{(2x-y)^3}}$

(3) $z_x = \dfrac{(x+y)_x}{x+y} = \dfrac{1}{x+y}$

$z_{xx} = \{(x+y)^{-1}\}_x = -(x+y)^{-2}(x+y)_x$

$= -\dfrac{1}{(x+y)^2}$

$z_{xy} = \{(x+y)^{-1}\}_y = -(x+y)^{-2}(x+y)_y$

$= -\dfrac{1}{(x+y)^2}$

(4) $z_x = (e^{3x}\cos 2y)_x = e^{3x}(3x)'\cos 2y$

$= 3e^{3x}\cos 2y$

$z_{xx} = (3e^{3x}\cos 2y)_x = 3e^{3x}(3x)'\cos 2y$

$= 9e^{3x}\cos 2y$

$z_{xy} = (3e^{3x}\cos 2y)_y = 3e^{3x}(-\sin 2y)(2y)'$

$= -6e^{3x}\sin 2y$

2. (1) $z_x = (3x^2 + 4xy - 2y^2)_x = 6x + 4y$

$z_{xx} = (6x+4y)_x = 6, z_{xy} = (6x+4y)_y = 4$

$z_y = (3x^2 + 4xy - 2y^2)_y = 4x - 4y$

$z_{yx} = (4x-4y)_x = 4, z_{yy} = (4x-4y)_y = -4$

(2) $z_x = (\sin x \sin y)_x = \cos x \sin y$

$z_{xx} = (\cos x \sin y)_x = -\sin x \sin y$

$z_{xy} = (\cos x \sin y)_y = \cos x \cos y$

$z_y = (\sin x \sin y)_y = \sin x \cos y$

$z_{yx} = (\sin x \cos y)_x = \cos x \cos y$

$z_{yy} = (\sin x \cos y)_y = -\sin x \sin y$