

解答

1. (1) 1次近似式 $\sqrt{x+1} = 1 + \frac{1}{2}x + \varepsilon_1$
 2次近似式 $\sqrt{x+1} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \varepsilon_2$
- (2) 1次近似式 $\frac{2}{1-x} = 2 + 2x + \varepsilon_1$
 2次近似式 $\frac{2}{1-x} = 2 + 2x + 2x^2 + \varepsilon_2$
2. (1) $e^{3x} = e^3 + 3e^3(x-1) + \frac{9e^3}{2}(x-1)^2 + \varepsilon_2$
 (2) $\frac{1}{x} = 1 - (x-1) + (x-1)^2 + \varepsilon_2$
3. (1) $\sin 4x = 4x - \frac{4^3}{3!}x^3 + \frac{4^5}{5!}x^5 + \dots$
 $+ \frac{(-1)^n 4^{2n+1}}{(2n+1)!}x^{2n+1} + \dots$
- (2) $\cos 3x = 1 - \frac{3^2}{2!}x^2 + \frac{3^4}{4!}x^4 + \dots$
 $+ \frac{(-1)^n 3^{2n}}{(2n)!}x^{2n} + \dots$

解説

1. (1) $f(x) = (x+1)^{\frac{1}{2}}, f(0) = \sqrt{1} = 1,$
 $f'(x) = \frac{1}{2}(x+1)^{-\frac{1}{2}}, f'(0) = \frac{1}{2}$
 $f''(x) = -\frac{1}{4}(x+1)^{-\frac{3}{2}}, f''(0) = -\frac{1}{4}$ を
 $f(x) = f(0) + f'(0)x + \varepsilon_1,$
 $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \varepsilon_2$ に代入
 する.
- (2) $f(x) = 2(1-x)^{-1}, f(0) = 2$
 $f'(x) = 2(1-x)^{-2}, f'(0) = 2$
 $f''(x) = 4(1-x)^{-3}, f''(0) = 4$ を
 $f(x) = f(0) + f'(0)x + \varepsilon_1,$
 $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \varepsilon_2$ に代入
 する.
2. (1) $f(x) = e^{3x}, f(1) = e^3,$
 $f'(x) = 3e^{3x}, f'(1) = 3e^3$
 $f''(x) = 9e^{3x}, f''(1) = 9e^3$ を
 $f(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \varepsilon_2$
 に代入する.
- (2) $f(x) = x^{-1}, f(1) = 1,$
 $f'(x) = -x^{-2}, f'(1) = -1$
 $f''(x) = 2x^{-3}, f''(1) = 2$ を
 $f(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \varepsilon_2$
 に代入する.
3. (1) 教科書 p21 (7) に $4x$ を代入する
 (2) 教科書 p21 (8) に $3x$ を代入する