第3章 3 「置換積分法」 第3回

解答

1. (1) $-\dfrac{1}{9}\cos^9 x + C$

(2) $\dfrac{1}{24}(4x+3)^6 + C$

(3) $\log(x^4+1) + C$

(4) $-\dfrac{1}{3}e^{-x^3} + C$

(5) $\log|e^x + \sin x| + C$

(6) $\log(4x^2+8x+5) + C$

2. (1) $\dfrac{5}{4}$ (2) $\dfrac{1}{160}$

(3) $\dfrac{15}{16}$ (4) $\log 2$

解説

1. (1) $\cos x = t$ とおくと,

$-\sin x dx = dt$ より $\sin x dx = -dt$

$\displaystyle\int \cos^8 x \sin x dx = -\int t^8 dt$

$= -\dfrac{1}{9}t^9 + C = -\dfrac{1}{9}\cos^9 x + C$

(2) $4x+3 = t$ とおくと,

$4dx = dt$ より $xdx = \dfrac{1}{4}dt$

$\displaystyle\int (4x+3)^5 dx = \frac{1}{4}\int t^5 dt$

$= \dfrac{1}{24}t^6 + C = \dfrac{1}{24}(4x+3)^6 + C$

(3) $x^4+1 = t$ とおくと,

$4x^3dx = dt$

$\displaystyle\int \frac{4x^3}{x^4+1}dx = \int \frac{1}{t}dt$

$= \log|t| + C = \log|x^4+1| + C$

$= \log(x^4+1) + C$

(4) $-x^3 = t$ とおくと,

$-3x^2dx = dt$ より $x^2dx = -\dfrac{1}{3}dt$

$\displaystyle\int x^2 e^{-x^3} dx = -\frac{1}{3}\int e^t dt$

$= -\dfrac{1}{3}e^t + C = -\dfrac{1}{3}e^{-x^3} + C$

(5) $e^x + \sin x = t$ とおくと,

$(e^x + \cos x)dx = dt$

$\displaystyle\int \frac{e^x+\cos x}{e^x+\sin x}dx = \int \frac{1}{t}dt$

$= \log|t| + C = \log|e^x + \sin x| + C$

(6) $4x^2+8x+5 = t$ とおくと,

$(8x+8)dx = dt$

$\displaystyle\int \frac{8x+8}{4x^2+8x+5}dx = \int \frac{1}{t}dt$

$= \log|t| + C = \log|4x^2+8x+5| + C$

$= \log(4x^2+8x+5) + C$

2. (1) $3x-1 = t$ とおくと,

$3dx = dt$ より $dx = \dfrac{1}{3}dt$

$\displaystyle\int_0^1 (3x-1)^3 dx = \frac{1}{3}\int_{-1}^2 t^3 dt = \frac{5}{4}$

(2) $\sin x = t$ とおくと,

$\cos x dx = dt$

$\displaystyle\int_0^{\frac{\pi}{6}} \sin^4 x \cos x dx = \int_0^{\frac{1}{2}} t^4 dt = \frac{1}{160}$

(3) $x^4+1 = t$ とおくと,

$4x^3dx = dt$ より $x^3dx = \dfrac{1}{4}dt$

$\displaystyle\int_0^1 x^3(x^4+1)^3 dx = \frac{1}{4}\int_1^2 t^3 dt = \frac{15}{16}$

(4) $x^2-x+2 = t$ とおくと,

$(2x-1)dx = dt$

$\displaystyle\int_1^2 \frac{2x-1}{x^2-x+2}dx = \int_2^4 \frac{1}{t}dt = \Big[\log|t|\Big]_2^4$

$= \log 4 - \log 2 = \log\dfrac{4}{2} = \log 2$