第 3 章 3 「置換積分法」 第 1 回

解答

1. (1) $\dfrac{1}{5}\sin^5 x + C$

(2) $\dfrac{1}{12}(2x+1)^6 + C$

(3) $\dfrac{1}{8}(x^2+2)^4 + C$

(4) $\dfrac{1}{2}e^{x^2} + C$

(5) $\log|e^x - 1| + C$

(6) $\log(x^2+1) + C$

2. (1) 10　(2) $\dfrac{1}{3}$

(3) $\dfrac{1}{3}$　(4) $\log 3$

解説

1. (1)~(4) は教科書 p101 例題 1，(5) と (6) は教科書 p102 例題 2 のように考える．

(1) $\sin x = t$ とおくと，

$(\sin x)'dx = dt$ より $\cos x dx = dt$

$$\int \sin^4 x \cos x dx = \int t^4 dt$$
$$= \frac{1}{5}t^5 + C = \frac{1}{5}\sin^5 x + C$$

(2) $2x+1 = t$ とおくと，

$2dx = dt$ より $dx = \dfrac{1}{2}dt$

$$\int (2x+1)^5 dx = \frac{1}{2}\int t^5 dt$$
$$= \frac{1}{12}t^6 + C = \frac{1}{12}(2x+1)^6 + C$$

(3) $x^2+2 = t$ とおくと，

$2xdx = dt$ より $xdx = \dfrac{1}{2}dt$

$$\int x(x^2+2)^3 dx = \frac{1}{2}\int t^3 dt$$
$$= \frac{1}{8}t^4 + C = \frac{1}{8}(x^2+2)^4 + C$$

(4) $x^2 = t$ とおくと，

$2xdx = dt$ より $xdx = \dfrac{1}{2}dt$

$$\int xe^{x^2} dx = \frac{1}{2}\int e^t dt$$
$$= \frac{1}{2}e^t + C = \frac{1}{2}e^{x^2} + C$$

(5) $e^x - 1 = t$ とおくと，

$e^x dx = dt$

$$\int \frac{e^x}{e^x - 1}dx = \int \frac{1}{t}dt$$
$$= \log|t| + C = \log|e^x - 1| + C$$

(6) $x^2 + 1 = t$ とおくと，

$2xdx = dt$

$$\int \frac{2x}{x^2+1}dx = \int \frac{1}{t}dt$$
$$= \log|t| + C = \log|x^2+1| + C$$
$$= \log(x^2+1) + C$$

2. 教科書 p103 例題 3 のように考える．

(1) $2x+1 = t$ とおくと，

$2dx = dt$ より $dx = \dfrac{1}{2}dt$

$$\int_0^1 (2x+1)^3 dx = \frac{1}{2}\int_1^3 t^3 dt = 10$$

(2) $\cos x = t$ とおくと，

$-\sin x dx = dt$ より $\sin x dx = -dt$

$$\int_0^{\frac{\pi}{2}} \cos^2 x \sin x dx = -\int_1^0 t^2 dt = \frac{1}{3}$$

(3) $\log x = t$ とおくと，

$\dfrac{1}{x}dx = dt$

$$\int_1^e \frac{(\log x)^2}{x}dx = \int_0^1 t^2 dt = \frac{1}{3}$$

(4) $x^2 + x + 1 = t$ とおくと，

$(2x+1)dx = dt$

$$\int_0^1 \frac{2x+1}{x^2+x+1}dx = \int_1^3 \frac{1}{t}dt = \Big[\log|t|\Big]_1^3$$
$$= \log 3 - \log 1 = \log 3$$