

第3章 1 「不定積分」 第3回

解答

1. (1) $\frac{1}{5}x^5 + C$

(2) $-\frac{1}{6x^6} + C$

(3) $\frac{2}{7}x^3\sqrt{x} + C$

(4) $3\sqrt[3]{x} + C$

2. (1) $-x^3 + x^2 + 6x + C$

(2) $\frac{1}{4}x^4 + 2x^3 + 4x^2 - 2x + C$

(3) $2\sin x - 3\cos x + C$

(4) $2e^x + 3\log|x| + C$

(5) $\frac{1}{5}x^5 - 2x^2 - \frac{4}{x} + C$

3. (1) $\frac{1}{12}(2x-3)^6 + C$

(2) $-\frac{1}{4}\cos 4x + C$

(3) $-\frac{1}{2}e^{-2x} + C$

(4) $\frac{1}{3}\sqrt{(2x-1)^3} + C$

解説

1. (1) $\int x^4 dx = \frac{1}{5}x^5 + C$

(2) $\int \frac{1}{x^7} dx = \int x^{-7} dx = -\frac{1}{6x^6} + C$

(3) $\int x\sqrt{x^3} dx = \int x^{\frac{5}{2}} dx$
 $= \frac{2}{7}x^{\frac{7}{2}} + C = \frac{2}{7}x^3\sqrt{x} + C$

(4) $\int \frac{1}{\sqrt[3]{x^2}} dx = \int x^{-\frac{2}{3}} dx$
 $= 3x^{\frac{1}{3}} + C = 3\sqrt[3]{x} + C$

2. (1) $\int (-3x^2 + 2x + 6) dx$
 $= -3 \int x^2 dx + 2 \int x dx + 6 \int 1 dx$
 $= -x^3 + x^2 + 6x + C$

(2) $\int (x^3 + 6x^2 + 8x - 2) dx$
 $= \int x^3 dx + 6 \int x^2 dx + 8 \int x dx - 2 \int 1 dx$
 $= \frac{1}{4}x^4 + 2x^3 + 4x^2 - 2x + C$

(3) $\int (2\cos x + 3\sin x) dx$
 $= 2 \int \cos x dx + 3 \int \sin x dx$
 $= 2\sin x - 3\cos x + C$

(4) $\int \left(2e^x + \frac{3}{x}\right) dx$
 $= 2 \int e^x dx + 3 \int \frac{1}{x} dx$
 $= 2e^x + 3\log|x| + C$

(5) $\int \left(x^2 - \frac{2}{x}\right)^2 dx$
 $= \int \left(x^4 - 4x + \frac{4}{x^2}\right) dx$
 $= \int x^4 dx - 4 \int x dx + 4 \int \frac{1}{x^2} dx$
 $= \frac{1}{5}x^5 - 2x^2 - \frac{4}{x} + C$

3. (1) $\int x^5 dx = \frac{1}{6}x^6 + C$ より
 $\int (2x-3)^5 dx = \frac{1}{2} \cdot \frac{1}{6}(2x-3)^6 + C$
 $= \frac{1}{12}(2x-3)^6 + C$

(2) $\int \sin x dx = -\cos x + C$ より
 $\int \sin 4x dx = -\frac{1}{4}\cos 4x + C$

(3) $\int e^x dx = e^x + C$ より
 $\int e^{-2x} dx = -\frac{1}{2}e^{-2x} + C$

(4) $\int \sqrt{x} dx = \frac{2}{3}x^{\frac{3}{2}} + C$ より
 $\int \sqrt{2x-1} dx$
 $= \frac{1}{2} \cdot \frac{2}{3}(2x-1)^{\frac{3}{2}} + C$
 $= \frac{1}{3}\sqrt{(2x-1)^3} + C$