第１章 7 「逆三角関数とその導関数」 第２回

解答

1. (1) $\frac{\pi}{3}$　(2) $-\frac{\pi}{2}$　(3) 0

(4) $\frac{5}{6}\pi$　(5) $\frac{\pi}{4}$　(6) $-\frac{\pi}{6}$

2. (1) $y' = \frac{2}{\sqrt{1-4x^2}}$　(2) $y' = \frac{1}{\sqrt{2-x^2}}$

(3) $y' = -\frac{4}{\sqrt{1-16x^2}}$　(4) $y' = -\frac{1}{\sqrt{4-x^2}}$

(5) $y' = \frac{3}{1+9x^2}$　(6) $y' = -\frac{1}{1+x^2}$

解説

1. 逆三角関数の定義式

$y = \sin^{-1} x \quad \Leftrightarrow \quad \sin y = x \quad \left(-\frac{\pi}{2} \leqq y \leqq \frac{\pi}{2}\right)$

$y = \cos^{-1} x \quad \Leftrightarrow \quad \cos y = x \quad (0 \leqq y \leqq \pi)$

$y = \tan^{-1} x \quad \Leftrightarrow \quad \tan y = x \quad \left(-\frac{\pi}{2} < y < \frac{\pi}{2}\right)$

を利用して，値を求めればよい．

(1) $y = \sin^{-1} \frac{\sqrt{3}}{2} \quad \Leftrightarrow \quad \sin y = \frac{\sqrt{3}}{2}$

これから $y = \frac{\pi}{3}$ となる．

よって $\sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$

(2) $y = \sin^{-1}(-1) \quad \Leftrightarrow \quad \sin y = -1$

これから $y = -\frac{\pi}{2}$ となる．

よって $\sin^{-1}(-1) = -\frac{\pi}{2}$

(3) $y = \cos^{-1} 1 \quad \Leftrightarrow \quad \cos y = 1$

これから $y = 0$ となる．

よって $\cos^{-1} 1 = 0$

(4) $y = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) \quad \Leftrightarrow \quad \cos y = -\frac{\sqrt{3}}{2}$

これから $y = \frac{5}{6}\pi$ となる．

よって $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5}{6}\pi$

(5) $y = \tan^{-1} 1 \quad \Leftrightarrow \quad \tan y = 1$

これから $y = \frac{\pi}{4}$ となる．

よって $\tan^{-1} 1 = \frac{\pi}{4}$

(6) $y = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) \quad \Leftrightarrow \quad \tan y = -\frac{1}{\sqrt{3}}$

これから $y = -\frac{\pi}{6}$ となる．

よって $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

2. (1) $y = \sin^{-1} u, \quad u = 2x$

$\frac{dy}{du} = \frac{1}{\sqrt{1-u^2}}, \quad \frac{du}{dx} = 2$　より

$$y' = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{2}{\sqrt{1-u^2}}$$
$$= \frac{2}{\sqrt{1-(2x)^2}} = \frac{2}{\sqrt{1-4x^2}}$$

(2) $y = \sin^{-1} u, \quad u = \frac{x}{\sqrt{2}}$

$\frac{dy}{du} = \frac{1}{\sqrt{1-u^2}}, \quad \frac{du}{dx} = \frac{1}{\sqrt{2}}$　より

$$y' = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1-u^2}}$$
$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{2}}\right)^2}} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1-\frac{x^2}{2}}}$$
$$= \frac{1}{\sqrt{2-x^2}}$$

(3) $y = \cos^{-1} u, \quad u = 4x$

$\frac{dy}{du} = -\frac{1}{\sqrt{1-u^2}}, \quad \frac{du}{dx} = 4$　より

$$y' = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = -\frac{4}{\sqrt{1-u^2}}$$
$$= -\frac{4}{\sqrt{1-(4x)^2}} = -\frac{4}{\sqrt{1-16x^2}}$$

(4) $y = \cos^{-1} u, \quad u = \frac{x}{2}$

$\frac{dy}{du} = -\frac{1}{\sqrt{1-u^2}}, \quad \frac{du}{dx} = \frac{1}{2}$　より

$$y' = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = -\frac{1}{2}\frac{1}{\sqrt{1-u^2}}$$
$$= -\frac{1}{2\sqrt{1-\left(\frac{x}{2}\right)^2}} = -\frac{1}{\sqrt{4-x^2}}$$

(5) $y = \tan^{-1} u, \quad u = 3x$

$\frac{dy}{du} = \frac{1}{1+u^2}, \quad \frac{du}{dx} = 3$　より

$$y' = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{3}{1+u^2}$$
$$= \frac{3}{1+(3x)^2} = \frac{3}{1+9x^2}$$

(6) $y = \tan^{-1} u, \quad u = -x$

$\frac{dy}{du} = \frac{1}{1+u^2}, \quad \frac{du}{dx} = -1$　より

$$y' = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = -\frac{1}{1+u^2}$$
$$= -\frac{1}{1+(-x)^2} = -\frac{1}{1+x^2}$$