第１章 6 「対数関数の性質を用いた微分法」 第２回

解答

1. (1) $y' = \dfrac{2x-1}{(x+1)(x-2)}$

(2) $y' = \dfrac{12x+1}{(2x+1)(3x-1)}$

(3) $y' = -\dfrac{3}{(x+1)(x-2)}$

(4) $y' = -\dfrac{5}{(2x+1)(3x-1)}$

(5) $y' = \dfrac{4x}{(x+1)(x-1)}$

(6) $y' = -\dfrac{x+5}{(x+1)(x-1)}$

(7) $y' = \dfrac{3x+2}{x(x+1)}$

(8) $y' = \dfrac{5x+4}{2x(x+1)}$

解説

1. (1) $y = \log(x+1) + \log(x-2)$

$$y' = \frac{1}{x+1} + \frac{1}{x-2} = \frac{x-2}{(x+1)(x-2)} + \frac{x+1}{(x+1)(x-2)} = \frac{2x-1}{(x+1)(x-2)}$$

(2) $y = \log(2x+1) + \log(3x-1)$

$$y' = \frac{2}{2x+1} + \frac{3}{3x-1} = \frac{2(3x-1)}{(2x+1)(3x-1)} + \frac{3(2x+1)}{(2x+1)(3x-1)} = \frac{12x+1}{(2x+1)(3x-1)}$$

(3) $y = \log(x+1) - \log(x-2)$

$$y' = \frac{1}{x+1} - \frac{1}{x-2} = \frac{x-2}{(x+1)(x-2)} - \frac{x+1}{(x+1)(x-2)} = -\frac{3}{(x+1)(x-2)}$$

(4) $y = \log(2x+1) - \log(3x-1)$

$$y' = \frac{2}{2x+1} - \frac{3}{3x-1} = \frac{2(3x-1)}{(2x+1)(3x-1)} - \frac{3(2x+1)}{(2x+1)(3x-1)} = -\frac{5}{(2x+1)(3x-1)}$$

(5) $y = 2\log(x+1) + 2\log(x-1)$

$$y' = 2\cdot\frac{1}{x+1} + 2\cdot\frac{1}{x-1} = \frac{2(x-1)}{(x+1)(x-1)} + \frac{2(x+1)}{(x+1)(x-1)} = \frac{4x}{(x+1)(x-1)}$$

(6) $y = 2\log(x+1) - 3\log(x-1)$

$$y' = 2\cdot\frac{1}{x+1} - 3\cdot\frac{1}{x-1} = \frac{2(x-1)}{(x+1)(x-1)} - \frac{3(x+1)}{(x+1)(x-1)} = \frac{-x-5}{(x+1)(x-1)} = -\frac{x+5}{(x+1)(x-1)}$$

(7) $y = 2\log x + \log(x+1)$

$$y' = 2\cdot\frac{1}{x} + \frac{1}{x+1} = \frac{2(x+1)}{x(x+1)} + \frac{x}{x(x+1)} = \frac{3x+2}{x(x+1)}$$

(8) $y = 2\log x + \dfrac{1}{2}\log(x+1)$

$$y' = 2\cdot\frac{1}{x} + \frac{1}{2}\cdot\frac{1}{x+1} = \frac{4(x+1)}{2x(x+1)} + \frac{x}{2x(x+1)} = \frac{5x+4}{2x(x+1)}$$