

第2章 4 「逆行列と連立1次方程式」 第3回

解答

1. (1) $\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$ (2) $\begin{pmatrix} 13 & -2 & -2 \\ -4 & 1 & 0 \\ -6 & 1 & 1 \end{pmatrix}$

2. (1) $x = 3, y = 1, z = -1$ (2) $x = 8, y = -2, z = -3$

解説

1.

(1) $\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array} \right) \xrightarrow{2\text{行}-1\text{行}\times 2} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array} \right) \xrightarrow{1\text{行}-2\text{行}\times 2} \left(\begin{array}{cc|cc} 1 & 0 & 5 & -2 \\ 0 & 1 & -2 & 1 \end{array} \right)$

よって、逆行列は $\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$

(2) $\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 4 & 1 & 8 & 0 & 1 & 0 \\ 2 & -1 & 5 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{2\text{行}-1\text{行}\times 4 \\ 3\text{行}-1\text{行}\times 2}} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & -1 & 1 & -2 & 0 & 1 \end{array} \right) \xrightarrow{3\text{行}+2\text{行}} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & 0 & 1 & -6 & 1 & 1 \end{array} \right)$

$\xrightarrow{1\text{行}-3\text{行}\times 2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 13 & -2 & -2 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & 0 & 1 & -6 & 1 & 1 \end{array} \right)$ よって、逆行列は $\begin{pmatrix} 13 & -2 & -2 \\ -4 & 1 & 0 \\ -6 & 1 & 1 \end{pmatrix}$

2. (1) 係数行列の逆行列を求める.

$\left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 2 & 5 & 5 & 0 & 1 & 0 \\ 3 & 4 & 5 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{2\text{行}-1\text{行}\times 2 \\ 3\text{行}-1\text{行}\times 3}} \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & -2 & -1 & -3 & 0 & 1 \end{array} \right) \xrightarrow{3\text{行}+2\text{行}\times 2} \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -7 & 2 & 1 \end{array} \right)$

$\xrightarrow{\substack{1\text{行}-3\text{行}\times 2 \\ 2\text{行}-1\text{行}\times 1}} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 15 & -4 & -2 \\ 0 & 1 & 0 & 5 & -1 & -1 \\ 0 & 0 & 1 & -7 & 2 & 1 \end{array} \right) \xrightarrow{1\text{行}-2\text{行}\times 2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & -2 & 0 \\ 0 & 1 & 0 & 5 & -1 & -1 \\ 0 & 0 & 1 & -7 & 2 & 1 \end{array} \right)$

よって、逆行列は $\begin{pmatrix} 5 & -2 & 0 \\ 5 & -1 & -1 \\ -7 & 2 & 1 \end{pmatrix}$ 求める解は $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 & -2 & 0 \\ 5 & -1 & -1 \\ -7 & 2 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 6 \\ 8 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$

(2) 係数行列の逆行列を求める.

$\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 2 & 3 & 3 & 0 & 1 & 0 \\ 3 & -1 & 9 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{2\text{行}-1\text{行}\times 2 \\ 3\text{行}-1\text{行}\times 3}} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & -4 & 3 & -3 & 0 & 1 \end{array} \right) \xrightarrow{3\text{行}+2\text{行}\times 4} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & -11 & 4 & 1 \end{array} \right)$

$\xrightarrow{3\text{行}\times (-1)} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 11 & -4 & -1 \end{array} \right) \xrightarrow{\substack{1\text{行}-3\text{行}\times 2 \\ 2\text{行}+3\text{行}\times 1}} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & -21 & 8 & 2 \\ 0 & 1 & 0 & 9 & -3 & -1 \\ 0 & 0 & 1 & 11 & -4 & -1 \end{array} \right) \xrightarrow{1\text{行}-2\text{行}\times 1}$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -30 & 11 & 3 \\ 0 & 1 & 0 & 9 & -3 & -1 \\ 0 & 0 & 1 & 11 & -4 & -1 \end{array} \right)$

よって、逆行列は $\begin{pmatrix} -30 & 11 & 3 \\ 9 & -3 & -1 \\ 11 & -4 & -1 \end{pmatrix}$ 求める解は $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -30 & 11 & 3 \\ 9 & -3 & -1 \\ 11 & -4 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \\ -3 \end{pmatrix}$