

## 第2章 4 「逆行列と連立1次方程式」 第2回

解答

1. (1)  $\begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}$

(2)  $\begin{pmatrix} 5 & -2 & 0 \\ 5 & -1 & -1 \\ -7 & 2 & 1 \end{pmatrix}$

2. (1)  $x = 5, y = 0, z = -1$

(2)  $x = -18, y = -7, z = 10$

解説

1.

(1)  $\left( \begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array} \right) \xrightarrow{2\text{行}-1\text{行}\times 3} \left( \begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -3 & 1 \end{array} \right) \xrightarrow{1\text{行}-2\text{行}\times 2} \left( \begin{array}{cc|cc} 1 & 0 & 7 & -2 \\ 0 & 1 & -3 & 1 \end{array} \right)$

よって、逆行列は  $\begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}$

(2)  $\left( \begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 2 & 5 & 5 & 0 & 1 & 0 \\ 3 & 4 & 5 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{2\text{行}-1\text{行}\times 2 \\ 3\text{行}-1\text{行}\times 3}} \left( \begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & -2 & -1 & -3 & 0 & 1 \end{array} \right) \xrightarrow{3\text{行}+2\text{行}\times 2} \left( \begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -7 & 2 & 1 \end{array} \right)$

$\xrightarrow{\substack{1\text{行}-3\text{行}\times 2 \\ 2\text{行}-3\text{行}\times 1}} \left( \begin{array}{ccc|ccc} 1 & 2 & 0 & 15 & -4 & -2 \\ 0 & 1 & 0 & 5 & -1 & -1 \\ 0 & 0 & 1 & -7 & 2 & 1 \end{array} \right) \xrightarrow{1\text{行}-2\text{行}\times 2} \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & -2 & 0 \\ 0 & 1 & 0 & 5 & -1 & -1 \\ 0 & 0 & 1 & -7 & 2 & 1 \end{array} \right)$

よって、逆行列は  $\begin{pmatrix} 5 & -2 & 0 \\ 5 & -1 & -1 \\ -7 & 2 & 1 \end{pmatrix}$

2. (1) 係数行列の逆行列を求める.

$\left( \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 3 & 4 & 0 & 1 & 0 \\ 2 & 4 & 7 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{2\text{行}-1\text{行}\times 1 \\ 3\text{行}-1\text{行}\times 2}} \left( \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array} \right) \xrightarrow{\substack{1\text{行}-3\text{行}\times 3 \\ 2\text{行}-3\text{行}\times 1}} \left( \begin{array}{ccc|ccc} 1 & 2 & 0 & 7 & 0 & -3 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array} \right)$

$\xrightarrow{1\text{行}-2\text{行}\times 2} \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & -2 & -1 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array} \right)$

よって、逆行列は  $\begin{pmatrix} 5 & -2 & -1 \\ 1 & 1 & -1 \\ -2 & 0 & 1 \end{pmatrix}$  求める解は  $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 & -2 & -1 \\ 1 & 1 & -1 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}$

(2) 係数行列の逆行列を求める.

$\left( \begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 4 & 1 & 8 & 0 & 1 & 0 \\ 2 & -1 & 3 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{2\text{行}-1\text{行}\times 4 \\ 3\text{行}-1\text{行}\times 2}} \left( \begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array} \right) \xrightarrow{3\text{行}+2\text{行}\times 1} \left( \begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & 0 & -1 & -6 & 1 & 1 \end{array} \right)$

$\xrightarrow{3\text{行}\times (-1)} \left( \begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & 0 & 1 & 6 & -1 & -1 \end{array} \right) \xrightarrow{1\text{行}-3\text{行}\times 2} \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & -11 & 2 & 2 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & 0 & 1 & 6 & -1 & -1 \end{array} \right)$

よって、逆行列は  $\begin{pmatrix} -11 & 2 & 2 \\ -4 & 1 & 0 \\ 6 & -1 & -1 \end{pmatrix}$  求める解は  $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -11 & 2 & 2 \\ -4 & 1 & 0 \\ 6 & -1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -18 \\ -7 \\ 10 \end{pmatrix}$