第５章 4.「加法定理」 第５回

解答

1. (1) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$　(2) $-\dfrac{\sqrt{2}+\sqrt{6}}{4}$　(3) $-2+\sqrt{3}$

2. (1) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$　(2) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$　(3) $2+\sqrt{3}$

3. (1) $-\dfrac{56}{65}$　(2) $-\dfrac{63}{65}$　(3) $\dfrac{16}{63}$

解説

1. $\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$,　$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$,　$\tan(\alpha\pm\beta)=\dfrac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$

(1) $\sin 165^\circ=\sin(120^\circ+45^\circ)$
$=\sin 120^\circ\cos 45^\circ+\cos 120^\circ\sin 45^\circ$
$=\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}+\left(-\dfrac{1}{2}\right)\cdot\dfrac{\sqrt{2}}{2}=\dfrac{\sqrt{6}-\sqrt{2}}{4}$

(2) $\cos 165^\circ=\cos(120^\circ+45^\circ)$
$=\cos 120^\circ\cos 45^\circ-\sin 120^\circ\sin 45^\circ$
$=\left(-\dfrac{1}{2}\right)\cdot\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}=-\dfrac{\sqrt{2}+\sqrt{6}}{4}$

(3) $\tan 165^\circ=\tan(120^\circ+45^\circ)$
$=\dfrac{\tan 120^\circ+\tan 45^\circ}{1-\tan 120^\circ\tan 45^\circ}=\dfrac{-\sqrt{3}+1}{1-(-\sqrt{3})\cdot 1}=\dfrac{1-\sqrt{3}}{1+\sqrt{3}}=\dfrac{(1-\sqrt{3})^2}{(1+\sqrt{3})(1-\sqrt{3})}=\dfrac{1-2\sqrt{3}+3}{1-3}=\dfrac{4-2\sqrt{3}}{-2}$
$=-2+\sqrt{3}$

2. (1) $\sin 105^\circ=\sin(60^\circ+45^\circ)$
$=\sin 60^\circ\cos 45^\circ+\cos 60^\circ\sin 45^\circ$
$=\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}+\dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2}=\dfrac{\sqrt{6}+\sqrt{2}}{4}$

(2) $\cos 15^\circ=\cos(45^\circ-30^\circ)$
$=\cos 45^\circ\cos 30^\circ+\sin 45^\circ\sin 30^\circ$
$=\dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2}+\dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2}=\dfrac{\sqrt{6}+\sqrt{2}}{4}$

(3) $\tan 75^\circ=\tan(45^\circ+30^\circ)$
$=\dfrac{\tan 45^\circ+\tan 30^\circ}{1-\tan 45^\circ\tan 30^\circ}=\dfrac{1+\dfrac{1}{\sqrt{3}}}{1-1\cdot\dfrac{1}{\sqrt{3}}}=\dfrac{\sqrt{3}+1}{\sqrt{3}-1}=\dfrac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)}=\dfrac{3+2\sqrt{3}+1}{3-1}=\dfrac{4+2\sqrt{3}}{2}$
$=2+\sqrt{3}$

3. $\cos^2\alpha+\sin^2\alpha=1$ より，$\cos^2\alpha=1-\sin^2\alpha=1-\dfrac{9}{25}=\dfrac{16}{25}$ このとき，α は第３象限の角なので，$\cos\alpha<0$

よって $\cos\alpha=-\dfrac{4}{5}$　したがって, $\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}=-\dfrac{3}{5}\div\left(-\dfrac{4}{5}\right)=\dfrac{3}{5}\times\dfrac{5}{4}=\dfrac{3}{4}$

$\cos^2\beta+\sin^2\beta=1$ より，$\sin^2\beta=1-\cos^2\beta=1-\dfrac{144}{169}=\dfrac{25}{169}$ このとき，β は第４象限の角なので，$\sin\beta<0$

よって $\sin\beta=-\dfrac{5}{13}$　したがって, $\tan\beta=\dfrac{\sin\beta}{\cos\beta}=-\dfrac{5}{13}\div\dfrac{12}{13}=-\dfrac{5}{13}\times\dfrac{13}{12}=-\dfrac{5}{12}$

$\cos\alpha=-\dfrac{4}{5}$，$\tan\alpha=\dfrac{3}{4}$，$\sin\beta=-\dfrac{5}{13}$，$\tan\beta=-\dfrac{5}{12}$

(1) $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$
$=\left(-\dfrac{3}{5}\right)\cdot\dfrac{12}{13}-\left(-\dfrac{4}{5}\right)\cdot\left(-\dfrac{5}{13}\right)=-\dfrac{36}{65}-\dfrac{20}{65}$
$=-\dfrac{56}{65}$

(2) $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$
$=\left(-\dfrac{4}{5}\right)\cdot\dfrac{12}{13}-\left(-\dfrac{3}{5}\right)\cdot\left(-\dfrac{5}{13}\right)=-\dfrac{48}{65}-\dfrac{15}{65}$
$=-\dfrac{63}{65}$

(3) $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\dfrac{\dfrac{3}{4}-\dfrac{5}{12}}{1-\dfrac{3}{4}\cdot\left(-\dfrac{5}{12}\right)}=\dfrac{\dfrac{36}{48}-\dfrac{20}{48}}{1+\dfrac{15}{48}}=\dfrac{36-20}{48+15}=\dfrac{16}{63}$