第5章 4.「加法定理」 第4回

解答

1. (1) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ (2) $\dfrac{\sqrt{2}+\sqrt{6}}{4}$ (3) $2-\sqrt{3}$

2. (1) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ (2) $\dfrac{\sqrt{2}-\sqrt{6}}{4}$ (3) $-2+\sqrt{3}$

3. (1) $\dfrac{63}{65}$ (2) $-\dfrac{56}{65}$ (3) $\dfrac{63}{16}$

解説

1. $\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$, $\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$, $\tan(\alpha\pm\beta)=\dfrac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$

(1) $\sin 15^\circ=\sin(60^\circ-45^\circ)$
$=\sin 60^\circ\cos 45^\circ-\cos 60^\circ\sin 45^\circ$
$=\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}-\dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2}=\dfrac{\sqrt{6}-\sqrt{2}}{4}$

(2) $\cos 15^\circ=\cos(60^\circ-45^\circ)$
$=\cos 60^\circ\cos 45^\circ+\sin 60^\circ\sin 45^\circ$
$=\dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}=\dfrac{\sqrt{2}+\sqrt{6}}{4}$

(3) $\tan 15^\circ=\tan(60^\circ-45^\circ)$
$=\dfrac{\tan 60^\circ-\tan 45^\circ}{1+\tan 60^\circ\tan 45^\circ}=\dfrac{\sqrt{3}-1}{1+\sqrt{3}\cdot 1}=\dfrac{\sqrt{3}-1}{1+\sqrt{3}}=\dfrac{(\sqrt{3}-1)(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}=\dfrac{\sqrt{3}-3-1+\sqrt{3}}{1-3}=\dfrac{2\sqrt{3}-4}{-2}$
$=-\sqrt{3}+2=2-\sqrt{3}$

2. (1) $\sin 75^\circ=\sin(45^\circ+30^\circ)$
$=\sin 45^\circ\cos 30^\circ+\cos 45^\circ\sin 30^\circ$
$=\dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2}+\dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2}=\dfrac{\sqrt{6}+\sqrt{2}}{4}$

(2) $\cos 105^\circ=\cos(60^\circ+45^\circ)$
$=\cos 60^\circ\cos 45^\circ-\sin 60^\circ\sin 45^\circ$
$=\dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}=\dfrac{\sqrt{2}-\sqrt{6}}{4}$

(3) $\tan 165^\circ=\tan(120^\circ+45^\circ)$
$=\dfrac{\tan 120^\circ+\tan 45^\circ}{1-\tan 120^\circ\tan 45^\circ}=\dfrac{-\sqrt{3}+1}{1-(-\sqrt{3})\cdot 1}=\dfrac{1-\sqrt{3}}{1+\sqrt{3}}=\dfrac{(1-\sqrt{3})^2}{(1+\sqrt{3})(1-\sqrt{3})}=\dfrac{1-2\sqrt{3}+3}{1-3}=\dfrac{4-2\sqrt{3}}{-2}$
$=-2+\sqrt{3}$

3. $\cos^2\alpha+\sin^2\alpha=1$ より，$\sin^2\alpha=1-\cos^2\alpha=1-\dfrac{16}{25}=\dfrac{9}{25}$ このとき，α は第4象限の角なので，$\sin\alpha<0$

よって $\sin\alpha=-\dfrac{3}{5}$ したがって, $\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}=-\dfrac{3}{5}\div\dfrac{4}{5}=-\dfrac{3}{5}\times\dfrac{5}{4}=-\dfrac{3}{4}$

$\cos^2\beta+\sin^2\beta=1$ より，$\cos^2\beta=1-\sin^2\beta=1-\dfrac{144}{169}=\dfrac{25}{169}$ このとき，β は第2象限の角なので，$\cos\beta<0$

よって $\cos\beta=-\dfrac{5}{13}$ したがって, $\tan\beta=\dfrac{\sin\beta}{\cos\beta}=\dfrac{12}{13}\div\left(-\dfrac{5}{13}\right)=-\dfrac{12}{13}\times\dfrac{13}{5}=-\dfrac{12}{5}$

(1) $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$
$=\left(-\dfrac{3}{5}\right)\cdot\left(-\dfrac{5}{13}\right)+\dfrac{4}{5}\cdot\dfrac{12}{13}=\dfrac{15}{65}+\dfrac{48}{65}=\dfrac{63}{65}$

(2) $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
$=\dfrac{4}{5}\cdot\left(-\dfrac{5}{13}\right)+\left(-\dfrac{3}{5}\right)\cdot\dfrac{12}{13}=-\dfrac{20}{65}-\dfrac{36}{65}=-\dfrac{56}{65}$

(3) $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\dfrac{-\dfrac{3}{4}-\dfrac{12}{5}}{1-\left(-\dfrac{3}{4}\right)\cdot\left(-\dfrac{12}{5}\right)}=\dfrac{-\dfrac{15}{20}-\dfrac{48}{20}}{1-\dfrac{36}{20}}=\dfrac{-15-48}{20-36}=\dfrac{-63}{-16}=\dfrac{63}{16}$