# 第 5 章 4.「加法定理」 第 3 回

## 解答

**1.** (1) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ (2) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ (3) $2-\sqrt{3}$

**2.** (1) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ (2) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ (3) $-2-\sqrt{3}$

**3.** (1) $\dfrac{63}{65}$ (2) $-\dfrac{16}{65}$ (3) $-\dfrac{33}{56}$

## 解説

**1.** $\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$, $\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$, $\tan(\alpha\pm\beta)=\dfrac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$

(1) $\sin 15^\circ=\sin(45^\circ-30^\circ)$
$=\sin 45^\circ\cos 30^\circ-\cos 45^\circ\sin 30^\circ$
$=\dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2}-\dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2}=\dfrac{\sqrt{6}-\sqrt{2}}{4}$

(2) $\cos 15^\circ=\cos(45^\circ-30^\circ)$
$=\cos 45^\circ\cos 30^\circ+\sin 45^\circ\sin 30^\circ$
$=\dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2}+\dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2}=\dfrac{\sqrt{6}+\sqrt{2}}{4}$

(3) $\tan 15^\circ=\tan(45^\circ-30^\circ)$

$$=\frac{\tan 45^\circ-\tan 30^\circ}{1+\tan 45^\circ\tan 30^\circ}=\frac{1-\dfrac{1}{\sqrt{3}}}{1+1\cdot\dfrac{1}{\sqrt{3}}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}=\frac{3-2\sqrt{3}+1}{3-1}=\frac{4-2\sqrt{3}}{2}$$

$=2-\sqrt{3}$

**2.** (1) $\sin 165^\circ=\sin(120^\circ+45^\circ)$
$=\sin 120^\circ\cos 45^\circ+\cos 120^\circ\sin 45^\circ$
$=\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}+\left(-\dfrac{1}{2}\right)\cdot\dfrac{\sqrt{2}}{2}=\dfrac{\sqrt{6}-\sqrt{2}}{4}$

(2) $\cos 75^\circ=\cos(45^\circ+30^\circ)$
$=\cos 45^\circ\cos 30^\circ-\sin 45^\circ\sin 30^\circ$
$=\dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2}-\dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2}=\dfrac{\sqrt{6}-\sqrt{2}}{4}$

(3) $\tan 105^\circ=\tan(60^\circ+45^\circ)$

$$=\frac{\tan 60^\circ+\tan 45^\circ}{1-\tan 60^\circ\tan 45^\circ}=\frac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1}=\frac{1+\sqrt{3}}{1-\sqrt{3}}=\frac{(1+\sqrt{3})^2}{(1-\sqrt{3})(1+\sqrt{3})}=\frac{3+2\sqrt{3}+1}{1-3}=\frac{4+2\sqrt{3}}{-2}$$

$=-2-\sqrt{3}$

**3.** $\cos^2\alpha+\sin^2\alpha=1$ より，$\sin^2\alpha=1-\cos^2\alpha=1-\dfrac{9}{25}=\dfrac{16}{25}$ このとき，$\alpha$ は第 2 象限の角なので，$\sin\alpha>0$

よって $\sin\alpha=\dfrac{4}{5}$ したがって, $\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}=\dfrac{4}{5}\div\left(-\dfrac{3}{5}\right)=-\dfrac{4}{5}\times\dfrac{5}{3}=-\dfrac{4}{3}$

$\cos^2\beta+\sin^2\beta=1$ より，$\cos^2\beta=1-\sin^2\beta=1-\dfrac{25}{169}=\dfrac{144}{169}$ このとき，$\beta$ は第 4 象限の角なので，$\cos\beta>0$

よって $\cos\beta=\dfrac{12}{13}$ したがって, $\tan\beta=\dfrac{\sin\beta}{\cos\beta}=-\dfrac{5}{13}\div\dfrac{12}{13}=-\dfrac{5}{13}\times\dfrac{13}{12}=-\dfrac{5}{12}$

(1) $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$
$=\dfrac{4}{5}\cdot\dfrac{12}{13}+\left(-\dfrac{3}{5}\right)\cdot\left(-\dfrac{5}{13}\right)=\dfrac{48}{65}+\dfrac{15}{65}=\dfrac{63}{65}$

(2) $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$
$=\left(-\dfrac{3}{5}\right)\cdot\dfrac{12}{13}-\dfrac{4}{5}\cdot\left(-\dfrac{5}{13}\right)=-\dfrac{36}{65}+\dfrac{20}{65}=-\dfrac{16}{65}$

(3) $$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}=\frac{-\dfrac{4}{3}-\left(-\dfrac{5}{12}\right)}{1+\left(-\dfrac{4}{3}\right)\cdot\left(-\dfrac{5}{12}\right)}=\frac{-\dfrac{48}{36}+\dfrac{15}{36}}{1+\dfrac{20}{36}}=\frac{-48+15}{36+20}=-\frac{33}{56}$$