第５章 2.「鈍角の三角比」 第５回

解答

1. (1) $\dfrac{1}{\sqrt{2}}$ または $\dfrac{\sqrt{2}}{2}$　(2) $-\dfrac{1}{2}$　(3) $-\sqrt{3}$　(4) $\dfrac{1}{2}$

(5) $-\dfrac{1}{\sqrt{2}}$ または $-\dfrac{\sqrt{2}}{2}$　(6) $-\dfrac{1}{\sqrt{3}}$ または $-\dfrac{\sqrt{3}}{3}$　(7) 1　(8) -1

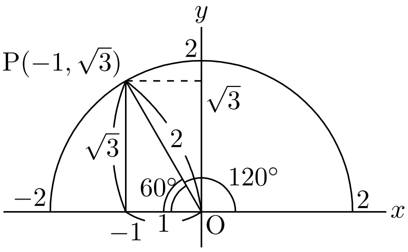
2. (1) $\sin 72^\circ$　(2) $-\cos 25^\circ$　(3) $-\tan 51^\circ$

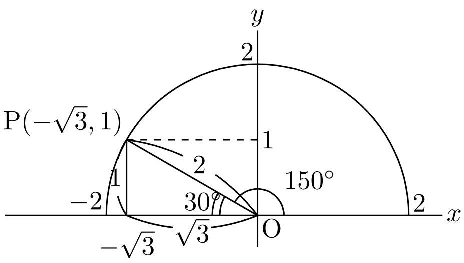
3. (1) $-\dfrac{4}{5}$　(2) $-\dfrac{3}{4}$

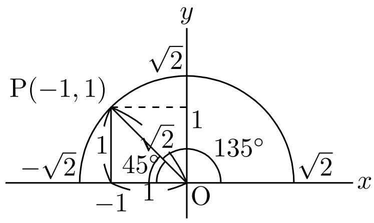
4. (1) $\dfrac{1}{3}$　(2) $\dfrac{2\sqrt{2}}{3}$

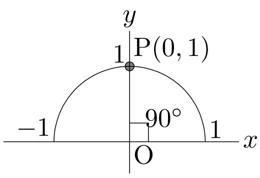
解説

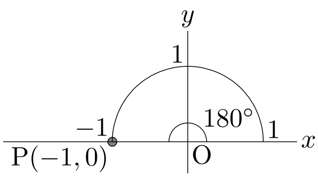
1. 原点を中心として半径 r の半円をかき，半円上の点 $\mathrm{P}(X, Y)$ とする．x 軸の正の向きと線分 OP のなす角を α とすると， $\sin\alpha = \dfrac{Y}{r}$，$\cos\alpha = \dfrac{X}{r}$，$\tan\alpha = \dfrac{Y}{X}$











(1) $\sin 135^\circ = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$　(2) $\cos 120^\circ = \dfrac{-1}{2} = -\dfrac{1}{2}$　(3) $\tan 120^\circ = \dfrac{\sqrt{3}}{-1} = -\sqrt{3}$

(4) $\sin 150^\circ = \dfrac{1}{2}$　(5) $\cos 135^\circ = \dfrac{-1}{\sqrt{2}} = -\dfrac{1}{\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$　(6) $\tan 150^\circ = \dfrac{1}{-\sqrt{3}} = -\dfrac{1}{\sqrt{3}} = -\dfrac{\sqrt{3}}{3}$

(7) $\sin 90^\circ = \dfrac{1}{1} = 1$　(8) $\cos 180^\circ = \dfrac{-1}{1} = -1$

2. (1) $72^\circ + 108^\circ = 180^\circ$ より，
$\sin 108^\circ = \sin 72^\circ$

(2) $25^\circ + 155^\circ = 180^\circ$ より，
$\cos 155^\circ = -\cos 25^\circ$

(3) $51^\circ + 129^\circ = 180^\circ$ より，
$\tan 129^\circ = -\tan 51^\circ$

3. (1) $\cos^2\alpha + \sin^2\alpha = 1$ より，$\cos^2\alpha = 1 - \sin^2\alpha = 1 - \dfrac{9}{25} = \dfrac{16}{25}$ このとき，α は鈍角なので，$\cos\alpha < 0$

よって $\cos\alpha = -\dfrac{4}{5}$

(2) $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{3}{5} \div \left(-\dfrac{4}{5}\right) = -\dfrac{3}{5} \times \dfrac{5}{4} = -\dfrac{3}{4}$

4. (1) $1 + \tan^2\alpha = \dfrac{1}{\cos^2\alpha}$ より，$\dfrac{1}{\cos^2\alpha} = 1 + \tan^2\alpha = 1 + 8 = 9$ よって $\cos^2\alpha = \dfrac{1}{9}$

α は鋭角なので，$\cos\alpha > 0$ よって $\cos\alpha = \dfrac{1}{3}$

(2) $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$ より，$\sin\alpha = \tan\alpha\cos\alpha = 2\sqrt{2} \cdot \dfrac{1}{3} = \dfrac{2\sqrt{2}}{3}$