第３章 6 「極座標による２重積分（その３）」 第３回

解答

1. (1) $\dfrac{56}{3}$

(2) 3

(3) $\dfrac{8}{3}$

(4) $\dfrac{2}{11}$

(5) $\dfrac{7}{24}$

解説

1. (1) $\displaystyle\iint_D y dxdy$

$$= \int_0^{\frac{\pi}{2}}\int_2^4 r\sin\theta\, rdrd\theta$$
$$= \int_0^{\frac{\pi}{2}}\left\{\int_2^4 r^2\sin\theta dr\right\}d\theta$$
$$= \int_0^{\frac{\pi}{2}}\left\{\int_2^4 r^2 dr\right\}\sin\theta\, d\theta$$
$$= \int_0^{\frac{\pi}{2}}\left[\frac{1}{3}r^3\right]_2^4\sin\theta\, d\theta$$
$$= \frac{56}{3}\int_0^{\frac{\pi}{2}}\sin\theta\, d\theta$$
$$= \frac{56}{3}\Big[-\cos\theta\Big]_0^{\frac{\pi}{2}}$$
$$= \frac{56}{3}\{-0-(-1)\}$$
$$= \frac{56}{3}$$

(2) $\displaystyle\iint_D \frac{y}{x^2+y^2}dxdy$

$$= \int_0^{\frac{\pi}{2}}\int_1^4 \frac{r\sin\theta}{r^2}rdrd\theta$$
$$= \int_0^{\frac{\pi}{2}}\left\{\int_1^4 \sin\theta dr\right\}d\theta$$
$$= \int_0^{\frac{\pi}{2}}\left\{\int_1^4 dr\right\}\sin\theta\, d\theta$$
$$= \int_0^{\frac{\pi}{2}}\Big[\ r\ \Big]_1^4\sin\theta\, d\theta$$
$$= 3\int_0^{\frac{\pi}{2}}\sin\theta\, d\theta$$
$$= 3\Big[-\cos\theta\Big]_0^{\frac{\pi}{2}}$$
$$= 3\{-0-(-1)\}$$
$$= 3$$

(3) $\displaystyle\iint_D x\left(x^2+y^2\right)\sqrt{x^2+y^2}dxdy$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{\sqrt{2}} r\cos\theta\, r^2\sqrt{r^2}rdrd\theta$$
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left\{\int_0^{\sqrt{2}} r^5\cos\theta dr\right\}d\theta$$
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left\{\int_0^{\sqrt{2}} r^5 dr\right\}\cos\theta\, d\theta$$
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[\frac{1}{6}r^6\right]_0^{\sqrt{2}}\cos\theta\, d\theta$$
$$= \frac{4}{3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\theta d\theta$$
$$= \frac{4}{3}\Big[\ \sin\theta\ \Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$
$$= \frac{4}{3}\{1-(-1)\}$$
$$= \frac{8}{3}$$

(4) $\displaystyle\iint_D y\left(x^2+y^2\right)^4dxdy$

$$= \int_0^{\pi}\int_0^1 r\sin\theta\,(r^2)^4rdrd\theta$$
$$= \int_0^{\pi}\left\{\int_0^1 r^{10}\sin\theta dr\right\}d\theta$$
$$= \int_0^{\pi}\left\{\int_0^1 r^{10}dr\right\}\sin\theta\, d\theta$$
$$= \int_0^{\pi}\left[\frac{1}{11}r^{11}\right]_0^1\sin\theta\, d\theta$$
$$= \frac{1}{11}\int_0^{\pi}\sin\theta d\theta$$
$$= \frac{1}{11}\Big[\ -\cos\theta\ \Big]_0^{\pi}$$
$$= \frac{1}{11}\{-(-1)-(-1)\}$$
$$= \frac{2}{11}$$

(5) $\displaystyle\iint_D \frac{x}{\left(x^2+y^2\right)^3}dxdy$

$$= \int_0^{\frac{\pi}{2}}\int_1^2 \frac{r\cos\theta}{(r^2)^3}rdrd\theta$$
$$= \int_0^{\frac{\pi}{2}}\left\{\int_1^2 \frac{1}{r^4}\cos\theta dr\right\}d\theta$$
$$= \int_0^{\frac{\pi}{2}}\left\{\int_1^2 \frac{1}{r^4}dr\right\}\cos\theta\, d\theta$$
$$= \int_0^{\frac{\pi}{2}}\left[-\frac{1}{3}\frac{1}{r^3}\right]_1^2\cos\theta\, d\theta$$
$$= \left(-\frac{1}{24}+\frac{1}{3}\right)\int_0^{\frac{\pi}{2}}\cos\theta\, d\theta$$
$$= \frac{7}{24}\Big[\sin\theta\Big]_0^{\frac{\pi}{2}}$$
$$= \frac{7}{24}(1-0)$$
$$= \frac{7}{24}$$