第3章 5 「極座標による2重積分（その2）」 第2回

解答

1. (1) $\dfrac{49}{4}\pi$

(2) $\dfrac{81}{16}\pi$

(3) $\dfrac{8}{3}\pi$

(4) $\dfrac{3}{8}\pi$

(5) $\pi\log\dfrac{4}{3}$

解説

1. (1) $\displaystyle\iint_D dxdy$

$$
\begin{aligned}
&= \int_0^{\frac{\pi}{2}}\int_0^7 rdrd\theta \\
&= \int_0^{\frac{\pi}{2}}\left\{\int_0^7 rdr\right\}d\theta \\
&= \int_0^{\frac{\pi}{2}}\left[\frac{1}{2}r^2\right]_0^7 d\theta \\
&= \frac{49}{2}\int_0^{\frac{\pi}{2}} d\theta \\
&= \frac{49}{2}\Big[\,\theta\,\Big]_0^{\frac{\pi}{2}} \\
&= \frac{49}{4}\pi
\end{aligned}
$$

(2) $\displaystyle\iint_D \left(x^2+y^2\right)^3 dxdy$

$$
\begin{aligned}
&= \int_0^{\frac{\pi}{2}}\int_0^{\sqrt{3}} (r^2)^3 rdrd\theta \\
&= \int_0^{\frac{\pi}{2}}\left\{\int_0^{\sqrt{3}} r^7dr\right\}d\theta \\
&= \int_0^{\frac{\pi}{2}}\left[\frac{1}{8}r^8\right]_0^{\sqrt{3}} d\theta \\
&= \frac{81}{8}\int_0^{\frac{\pi}{2}} d\theta \\
&= \frac{81}{8}\Big[\,\theta\,\Big]_0^{\frac{\pi}{2}} \\
&= \frac{81}{16}\pi
\end{aligned}
$$

(3) $\displaystyle\iint_D \sqrt{x^2+y^2}dxdy$

$$
\begin{aligned}
&= \int_0^{\pi}\int_0^2 \sqrt{r^2}\,rdrd\theta \\
&= \int_0^{\pi}\left\{\int_0^2 r^2dr\right\}d\theta \\
&= \int_0^{\pi}\left[\frac{1}{3}r^3\right]_0^2 d\theta \\
&= \frac{8}{3}\int_0^{\pi} d\theta \\
&= \frac{8}{3}\Big[\,\theta\,\Big]_0^{\pi} \\
&= \frac{8}{3}\pi
\end{aligned}
$$

(4) $\displaystyle\iint_D \frac{1}{\left(x^2+y^2\right)^2}dxdy$

$$
\begin{aligned}
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_1^2 \frac{1}{(r^2)^2}rdrd\theta \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left\{\int_1^2 \frac{1}{r^3}dr\right\}d\theta \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[-\frac{1}{2}\frac{1}{r^2}\right]_1^2 d\theta \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(-\frac{1}{8}+\frac{1}{2}\right)d\theta \\
&= \frac{3}{8}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \\
&= \frac{3}{8}\Big[\,\theta\,\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= \frac{3}{8}\pi
\end{aligned}
$$

(5) $\displaystyle\iint_D \frac{1}{x^2+y^2}dxdy$

$$
\begin{aligned}
&= \int_0^{\pi}\int_3^4 \frac{1}{r^2}rdrd\theta \\
&= \int_0^{\pi}\left\{\int_3^4 \frac{1}{r}dr\right\}d\theta \\
&= \int_0^{\pi}\Big[\log|r|\Big]_3^4 d\theta \\
&= \int_0^{\pi}(\log 4-\log 3)\,d\theta \\
&= \log\frac{4}{3}\int_0^{\pi} d\theta \\
&= \log\frac{4}{3}\Big[\,\theta\,\Big]_0^{\pi} \\
&= \pi\log\frac{4}{3}
\end{aligned}
$$