第３章 ５ 「極座標による２重積分（その２）」 第１回

解答

1. (1) $\dfrac{5}{4}\pi$

(2) 2π

(3) $\dfrac{\pi}{2}\log 2$

(4) $\dfrac{26}{3}\pi$

解説

1. (1) $\displaystyle\iint_D dxdy$

$$
\begin{aligned}
&= \int_0^{\frac{\pi}{2}}\int_2^3 r dr d\theta \\
&= \int_0^{\frac{\pi}{2}}\left\{\int_2^3 r dr\right\} d\theta \\
&= \int_0^{\frac{\pi}{2}}\left[\frac{1}{2}r^2\right]_2^3 d\theta \\
&= \int_0^{\frac{\pi}{2}}\left(\frac{9}{2}-\frac{4}{2}\right) d\theta \\
&= \frac{5}{2}\int_0^{\frac{\pi}{2}} d\theta \\
&= \frac{5}{2}\Big[\ \theta\ \Big]_0^{\frac{\pi}{2}} \\
&= \frac{5}{4}\pi
\end{aligned}
$$

(2) $\displaystyle\iint_D \left(x^2+y^2\right) dxdy$

$$
\begin{aligned}
&= \int_0^{\frac{\pi}{2}}\int_0^2 r^2 r dr d\theta \\
&= \int_0^{\frac{\pi}{2}}\left\{\int_0^2 r^3 dr\right\} d\theta \\
&= \int_0^{\frac{\pi}{2}}\left[\frac{1}{4}r^4\right]_0^2 d\theta \\
&= 4\int_0^{\frac{\pi}{2}} d\theta \\
&= 4\Big[\ \theta\ \Big]_0^{\frac{\pi}{2}} \\
&= 2\pi
\end{aligned}
$$

(3) $\displaystyle\iint_D \frac{1}{x^2+y^2} dxdy$

$$
\begin{aligned}
&= \int_0^{\frac{\pi}{2}}\int_1^2 \frac{1}{r^2} r dr d\theta \\
&= \int_0^{\frac{\pi}{2}}\left\{\int_1^2 \frac{1}{r} dr\right\} d\theta \\
&= \int_0^{\frac{\pi}{2}}\Big[\log|r|\Big]_1^2 d\theta \\
&= \int_0^{\frac{\pi}{2}}(\log 2-\log 1)\, d\theta \\
&= \log 2\int_0^{\frac{\pi}{2}} d\theta \\
&= \log 2\Big[\ \theta\ \Big]_0^{\frac{\pi}{2}} \\
&= \frac{\pi}{2}\log 2
\end{aligned}
$$

(4) $\displaystyle\iint_D \sqrt{x^2+y^2} dxdy$

$$
\begin{aligned}
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_1^3 \sqrt{r^2} r dr d\theta \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left\{\int_1^3 r^2 dr\right\} d\theta \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[\frac{1}{3}r^3\right]_1^3 d\theta \\
&= \frac{26}{3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \\
&= \frac{26}{3}\Big[\ \theta\ \Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= \frac{26}{3}\pi
\end{aligned}
$$