第3章 4 「極座標による2重積分（その1）」 第3回

解答

1. (1) 81π

(2) 128π

(3) $\dfrac{\pi}{5}$

(4) $\dfrac{64}{5}\pi$

(5) $\dfrac{3}{5}\pi$

解説

1. (1) $\displaystyle\iint_D dxdy$

$$
\begin{aligned}
&= \int_0^{2\pi}\int_0^9 r dr d\theta \\
&= \int_0^{2\pi}\left\{\int_0^9 r dr\right\} d\theta \\
&= \int_0^{2\pi}\left[\frac{1}{2}r^2\right]_0^9 d\theta \\
&= \frac{81}{2}\int_0^{2\pi} d\theta \\
&= \frac{81}{2}\Big[\ \theta\ \Big]_0^{2\pi} \\
&= 81\pi
\end{aligned}
$$

(2) $\displaystyle\iint_D \left(x^2+y^2\right) dxdy$

$$
\begin{aligned}
&= \int_0^{2\pi}\int_0^4 r^2 r dr d\theta \\
&= \int_0^{2\pi}\left\{\int_0^4 r^3 dr\right\} d\theta \\
&= \int_0^{2\pi}\left[\frac{1}{4}r^4\right]_0^4 d\theta \\
&= 64\int_0^{2\pi} d\theta \\
&= 64\Big[\ \theta\ \Big]_0^{2\pi} \\
&= 128\pi
\end{aligned}
$$

(3) $\displaystyle\iint_D \left(x^2+y^2\right)^4 dxdy$

$$
\begin{aligned}
&= \int_0^{2\pi}\int_0^1 (r^2)^4 r dr d\theta \\
&= \int_0^{2\pi}\left\{\int_0^1 r^9 dr\right\} d\theta \\
&= \int_0^{2\pi}\left[\frac{1}{10}r^{10}\right]_0^1 d\theta \\
&= \frac{1}{10}\int_0^{2\pi} d\theta \\
&= \frac{1}{10}\Big[\ \theta\ \Big]_0^{2\pi} \\
&= \frac{\pi}{5}
\end{aligned}
$$

(4) $\displaystyle\iint_D \left(x^2+y^2\right)\sqrt{x^2+y^2}\,dxdy$

$$
\begin{aligned}
&= \int_0^{2\pi}\int_0^2 r^2\sqrt{r^2} r dr d\theta \\
&= \int_0^{2\pi}\left\{\int_0^2 r^4 dr\right\} d\theta \\
&= \int_0^{2\pi}\left[\frac{1}{5}r^5\right]_0^2 d\theta \\
&= \frac{32}{5}\int_0^{2\pi} d\theta \\
&= \frac{32}{5}\Big[\ \theta\ \Big]_0^{2\pi} \\
&= \frac{64}{5}\pi
\end{aligned}
$$

(5) $\displaystyle\iint_D \sqrt[3]{(x^2+y^2)^2}\,dxdy$

$$
\begin{aligned}
&= \int_0^{2\pi}\int_0^1 \sqrt[3]{r^4}\, r dr d\theta \\
&= \int_0^{2\pi}\left\{\int_0^1 r^{\frac{7}{3}} dr\right\} d\theta \\
&= \int_0^{2\pi}\left[\ \frac{3}{10}r^{\frac{10}{3}}\ \right]_0^1 d\theta \\
&= \frac{3}{10}\int_0^{2\pi} d\theta \\
&= \frac{3}{10}\Big[\ \theta\ \Big]_0^{2\pi} \\
&= \frac{3}{5}\pi
\end{aligned}
$$