第3章 4 「極座標による2重積分（その1）」 第2回

解答

1. (1) 25π

(2) $\dfrac{81}{2}\pi$

(3) $\dfrac{8}{3}\pi$

(4) $\dfrac{16}{3}\pi$

(5) $\dfrac{3}{4}\pi$

解説

1. (1) $\displaystyle\iint_D dxdy$

$$= \int_0^{2\pi}\int_0^5 rdrd\theta$$
$$= \int_0^{2\pi}\left\{\int_0^5 rdr\right\}d\theta$$
$$= \int_0^{2\pi}\left[\frac{1}{2}r^2\right]_0^5 d\theta$$
$$= \frac{25}{2}\int_0^{2\pi}d\theta$$
$$= \frac{25}{2}\left[\ \theta\ \right]_0^{2\pi}$$
$$= 25\pi$$

(2) $\displaystyle\iint_D \left(x^2+y^2\right)dxdy$

$$= \int_0^{2\pi}\int_0^3 r^2 rdrd\theta$$
$$= \int_0^{2\pi}\left\{\int_0^3 r^3dr\right\}d\theta$$
$$= \int_0^{2\pi}\left[\frac{1}{4}r^4\right]_0^3 d\theta$$
$$= \frac{81}{4}\int_0^{2\pi}d\theta$$
$$= \frac{81}{4}\left[\ \theta\ \right]_0^{2\pi}$$
$$= \frac{81}{2}\pi$$

(3) $\displaystyle\iint_D \left(x^2+y^2\right)^2dxdy$

$$= \int_0^{2\pi}\int_0^{\sqrt{2}}(r^2)^2rdrd\theta$$
$$= \int_0^{2\pi}\left\{\int_0^{\sqrt{2}} r^5dr\right\}d\theta$$
$$= \int_0^{2\pi}\left[\frac{1}{6}r^6\right]_0^{\sqrt{2}} d\theta$$
$$= \frac{4}{3}\int_0^{2\pi}d\theta$$
$$= \frac{4}{3}\left[\ \theta\ \right]_0^{2\pi}$$
$$= \frac{8}{3}\pi$$

(4) $\displaystyle\iint_D \sqrt{x^2+y^2}dxdy$

$$= \int_0^{2\pi}\int_0^2 \sqrt{r^2}rdrd\theta$$
$$= \int_0^{2\pi}\left\{\int_0^2 r^2dr\right\}d\theta$$
$$= \int_0^{2\pi}\left[\frac{1}{3}r^3\right]_0^2 d\theta$$
$$= \frac{8}{3}\int_0^{2\pi}d\theta$$
$$= \frac{8}{3}\left[\ \theta\ \right]_0^{2\pi}$$
$$= \frac{16}{3}\pi$$

(5) $\displaystyle\iint_D \sqrt[3]{x^2+y^2}dxdy$

$$= \int_0^{2\pi}\int_0^1 \sqrt[3]{r^2}\,rdrd\theta$$
$$= \int_0^{2\pi}\left\{\int_0^1 r^{\frac{5}{3}}dr\right\}d\theta$$
$$= \int_0^{2\pi}\left[\ \frac{3}{8}r^{\frac{8}{3}}\ \right]_0^1 d\theta$$
$$= \frac{3}{8}\int_0^{2\pi}d\theta$$
$$= \frac{3}{8}\left[\ \theta\ \right]_0^{2\pi}$$
$$= \frac{3}{4}\pi$$