第3章 3 「2重積分の計算（その3）」 第3回

解答

1. (1) 1

(2) $\frac{1}{4}$

(3) $\frac{1}{6}$

(4) $\frac{1}{4}$

(5) $\frac{1}{8}$

(6) $\frac{\pi}{16}$

解説

1. (1) $\iint_D dxdy$

$$= \int_1^e \left\{ \int_0^{\frac{1}{y}} dx \right\} dy$$
$$= \int_1^e \left[\, x \, \right]_0^{\frac{1}{y}} dy$$
$$= \int_1^e \frac{1}{y} dy$$
$$= \left[\log|y| \right]_1^e$$
$$= \log e - \log 1$$
$$= 1$$

(2) $\iint_D \frac{y^3}{x^3} dxdy$

$$= \int_1^e \left\{ \int_0^{\sqrt{x}} \frac{y^3}{x^3} dy \right\} dx$$
$$= \int_1^e \left[\frac{1}{4} \cdot \frac{y^4}{x^3} \right]_0^{\sqrt{x}} dx$$
$$= \int_1^e \frac{1}{4} \cdot \frac{x^2}{x^3} dx$$
$$= \frac{1}{4} \int_1^e \frac{1}{x} dx$$
$$= \frac{1}{4} \left[\log|x| \right]_1^e$$
$$= \frac{1}{4} (\log e - \log 1)$$
$$= \frac{1}{4}$$

(3) $\iint_D y^4 dxdy$

$$= \int_0^1 \left\{ \int_0^y y^4 dx \right\} dy$$
$$= \int_0^1 \left[y^4 x \right]_0^y dy$$
$$= \int_0^1 y^5 dy$$
$$= \left[\frac{1}{6} y^6 \right]_0^1$$
$$= \frac{1}{6}$$

(4) $\iint_D ydxdy$

$$= \int_0^1 \left\{ \int_0^{1-y^2} ydx \right\} dy$$
$$= \int_0^1 \left[yx \right]_0^{1-y^2} dy$$
$$= \int_0^1 y(1-y^2)dy$$
$$= \int_0^1 (y-y^3)dy$$
$$= \left[\frac{1}{2}y^2 - \frac{1}{4}y^4 \right]_0^1$$
$$= \frac{1}{4}$$

(5) $\iint_D \frac{1}{y^2} dxdy$

$$= \int_1^2 \left\{ \int_{x^2}^{x^3} \frac{1}{y^2} dy \right\} dx$$
$$= \int_1^2 \left\{ \int_{x^2}^{x^3} y^{-2} dy \right\} dx$$
$$= \int_1^2 \left[-y^{-1} \right]_{x^2}^{x^3} dx$$
$$= \int_1^2 \left(-x^{-3} + x^{-2} \right) dx$$
$$= \left[\frac{1}{2}x^{-2} - x^{-1} \right]_1^2$$
$$= \left(\frac{1}{8} - \frac{1}{2} \right) - \left(\frac{1}{2} - 1 \right)$$
$$= \frac{1}{8}$$

(6) $\iint_D ydxdy$

$$= \int_0^{\frac{\pi}{4}} \left\{ \int_0^{\cos 2x} ydy \right\} dx$$
$$= \int_0^{\frac{\pi}{4}} \left[\frac{1}{2}y^2 \right]_0^{\cos 2x} dx$$
$$= \int_0^{\frac{\pi}{4}} \frac{1}{2} \cos^2 2x dx$$
$$= \int_0^{\frac{\pi}{4}} \frac{1}{2} \cdot \frac{1+\cos 4x}{2} dx$$
$$= \frac{1}{4} \int_0^{\frac{\pi}{4}} (1 + \cos 4x) \, dx$$
$$= \frac{1}{4} \left[x + \frac{1}{4} \sin 4x \right]_0^{\frac{\pi}{4}}$$
$$= \frac{1}{4} \left\{ \left(\frac{\pi}{4} + \frac{1}{4} \sin \pi \right) - \left(0 + \frac{1}{4} \sin 0 \right) \right\}$$
$$= \frac{\pi}{16}$$