第3章 3 「2重積分の計算（その3）」 第2回

解答

1. (1) e^3-1

(2) $\frac{1}{4}$

(3) $\frac{4}{15}$

(4) $\frac{13}{12}$

(5) $\frac{1}{2}$

(6) $\frac{\pi}{8}$

解説

1. (1) $\iint_D dxdy$

$$= \int_0^3 \left\{ \int_0^{e^x} dy \right\} dx$$
$$= \int_0^3 \Big[\ y\ \Big]_0^{e^x} dx$$
$$= \int_0^3 e^x dx$$
$$= \Big[e^x\Big]_0^3$$
$$= e^3 - 1$$

(2) $\iint_D xdxdy$

$$= \int_0^1 \left\{ \int_0^{\sqrt{y}} xdx \right\} dy$$
$$= \int_0^1 \left[\frac{1}{2}x^2\right]_0^{\sqrt{y}} dy$$
$$= \int_0^1 \frac{1}{2}ydy$$
$$= \left[\frac{1}{4}y^2\right]_0^1$$
$$= \frac{1}{4}$$

(3) $\iint_D \sqrt{x}dxdy$

$$= \int_0^1 \left\{ \int_0^{1-x} \sqrt{x}dy \right\} dx$$
$$= \int_0^1 \Big[\sqrt{x}y\Big]_0^{1-x} dx$$
$$= \int_0^1 \sqrt{x}(1-x)dx$$
$$= \int_0^1 \left(\sqrt{x} - x\sqrt{x}\right) dx$$
$$= \int_0^1 \left(x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) dx$$
$$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^1$$
$$= \frac{4}{15}$$

(4) $\iint_D ydxdy$

$$= \int_0^1 \left\{ \int_{y^2}^{y+2} ydx \right\} dy$$
$$= \int_0^1 \Big[yx\Big]_{y^2}^{y+2} dy$$
$$= \int_0^1 \left\{y(y+2) - y^3\right\} dy$$
$$= \int_0^1 \left(-y^3 + y^2 + 2y\right) dy$$
$$= \left[-\frac{1}{4}y^4 + \frac{1}{3}y^3 + y^2\right]_0^1$$
$$= \frac{13}{12}$$

(5) $\iint_D xdxdy$

$$= \int_0^1 \left\{ \int_{x^2}^{-x^2+2} xdy \right\} dx$$
$$= \int_0^1 \Big[xy\Big]_{x^2}^{-x^2+2} dx$$
$$= \int_0^1 \left\{x(-x^2+2) - x^3\right\} dx$$
$$= \int_0^1 \left(-2x^3 + 2x\right) dx$$
$$= \left[-\frac{1}{2}x^4 + x^2\right]_0^1$$
$$= \frac{1}{2}$$

(6) $\iint_D ydxdy$

$$= \int_0^{\frac{\pi}{2}} \left\{ \int_0^{\sin x} ydy \right\} dx$$
$$= \int_0^{\frac{\pi}{2}} \left[\frac{1}{2}y^2\right]_0^{\sin x} dx$$
$$= \int_0^{\frac{\pi}{2}} \frac{1}{2}\sin^2 xdx$$
$$= \int_0^{\frac{\pi}{2}} \frac{1}{2} \cdot \frac{1-\cos 2x}{2} dx$$
$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} (1-\cos 2x)\,dx$$
$$= \frac{1}{4}\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}$$
$$= \frac{1}{4}\left\{\left(\frac{\pi}{2} - \frac{1}{2}\sin\pi\right) - \left(0 - \frac{1}{2}\sin 0\right)\right\}$$
$$= \frac{\pi}{8}$$