

### 第3章 1 「2重積分の計算（その1）」 第3回

解答

1. (1) 60  
 (2) 6  
 (3) 24  
 (4) 2  
 (5) 36  
 (6)  $\frac{20}{3}$   
 (7)  $\frac{8}{3}$

解説

1. (1) 
$$\begin{aligned} \iint_D 2dx dy &= 2 \iint_D dx dy \\ &= 2 \int_0^5 \left\{ \int_0^6 dy \right\} dx \\ &= 2 \int_0^5 [y]_0^6 dx \\ &= 2 \int_0^5 6 dx = 12 \int_0^5 dx \\ &= 12 [x]_0^5 \\ &= 60 \end{aligned}$$

(2) 
$$\begin{aligned} \iint_D x dx dy &= \int_1^2 \left\{ \int_0^4 x dy \right\} dx \\ &= \int_1^2 [xy]_0^4 dx \\ &= \int_1^2 4x dx \\ &= [2x^2]_1^2 \\ &= 8 - 2 \\ &= 6 \end{aligned}$$

(3) 
$$\begin{aligned} \iint_D x^2 y^2 dx dy &= \int_0^2 \left\{ \int_0^3 x^2 y^2 dy \right\} dx \\ &= \int_0^2 \left[ \frac{1}{3} x^2 y^3 \right]_0^3 dx \\ &= \int_0^2 9x^2 dx \\ &= [3x^3]_0^2 \\ &= 24 \end{aligned}$$

(4) 
$$\begin{aligned} \iint_D 10x^4 y^3 dx dy &= \int_0^1 \left\{ \int_0^{\sqrt{2}} 10x^4 y^3 dy \right\} dx \\ &= \int_0^1 \left[ \frac{5}{2} x^4 y^4 \right]_0^{\sqrt{2}} dx \\ &= \int_0^1 10x^4 dx \\ &= [2x^5]_0^1 \\ &= 2 \end{aligned}$$

(5) 
$$\begin{aligned} \iint_D (2x + 3y) dx dy &= \int_0^3 \left\{ \int_0^2 (2x + 3y) dy \right\} dx \\ &= \int_0^3 \left[ 2xy + \frac{3}{2} y^2 \right]_0^2 dx \\ &= \int_0^3 (4x + 6) dx \\ &= [2x^2 + 6x]_0^3 \\ &= 36 \end{aligned}$$

(6) 
$$\begin{aligned} \iint_D (x^2 + y^2) dx dy &= \int_2^3 \left\{ \int_0^1 (x^2 + y^2) dy \right\} dx \\ &= \int_2^3 \left[ x^2 y + \frac{1}{3} y^3 \right]_0^1 dx \\ &= \int_2^3 \left( x^2 + \frac{1}{3} \right) dx \\ &= \left[ \frac{1}{3} x^3 + \frac{1}{3} x \right]_2^3 \\ &= (9 + 1) - \left( \frac{8}{3} + \frac{2}{3} \right) \\ &= \frac{20}{3} \end{aligned}$$

(7) 
$$\begin{aligned} \iint_D (x^2 y + xy^3) dx dy &= \int_0^1 \left\{ \int_0^2 (x^2 y + xy^3) dy \right\} dx \\ &= \int_0^1 \left[ \frac{1}{2} x^2 y^2 + \frac{1}{4} xy^4 \right]_0^2 dx \\ &= \int_0^1 (2x^2 + 4x) dx \\ &= \left[ \frac{2}{3} x^3 + 2x^2 \right]_0^1 \\ &= \frac{8}{3} \end{aligned}$$