

第3章 1 「2重積分の計算（その1）」 第2回

解答

1. (1) 1
 (2) 9
 (3) 36
 (4) 32
 (5) 8
 (6) 1
 (7) $\frac{21}{4}$

解説

1. (1)
$$\begin{aligned} \iint_D dx dy &= \int_0^1 \left\{ \int_0^1 dy \right\} dx \\ &= \int_0^1 [y]_0^1 dx \\ &= \int_0^1 dx \\ &= [x]_0^1 \\ &= 1 \end{aligned}$$

(2)
$$\begin{aligned} \iint_D xy dx dy &= \int_0^2 \left\{ \int_0^3 xy dy \right\} dx \\ &= \int_0^2 \left[\frac{1}{2} xy^2 \right]_0^3 dx \\ &= \int_0^2 \frac{9}{2} x dx \\ &= \left[\frac{9}{4} x^2 \right]_0^2 \\ &= 9 \end{aligned}$$

(3)
$$\begin{aligned} \iint_D x^3 y^2 dx dy &= \int_0^2 \left\{ \int_0^3 x^3 y^2 dy \right\} dx \\ &= \int_0^2 \left[\frac{1}{3} x^3 y^3 \right]_0^3 dx \\ &= \int_0^2 9x^3 dx \\ &= \left[\frac{9}{4} x^4 \right]_0^2 \\ &= 36 \end{aligned}$$

(4)
$$\begin{aligned} \iint_D 30x^4 y^5 dx dy &= \int_0^2 \left\{ \int_0^1 30x^4 y^5 dy \right\} dx \\ &= \int_0^2 [5x^4 y^6]_0^1 dx \\ &= \int_0^2 5x^4 dx \\ &= [x^5]_0^2 \\ &= 32 \end{aligned}$$

(5)
$$\begin{aligned} \iint_D (3x + 2y) dx dy &= \int_0^2 \left\{ \int_0^1 (3x + 2y) dy \right\} dx \\ &= \int_0^2 [3xy + y^2]_0^1 dx \\ &= \int_0^2 (3x + 1) dx \\ &= \left[\frac{3}{2} x^2 + x \right]_0^2 \\ &= 8 \end{aligned}$$

(6)
$$\begin{aligned} \iint_D (3x^2 - xy) dx dy &= \int_0^1 \left\{ \int_0^2 (3x^2 - xy) dy \right\} dx \\ &= \int_0^1 \left[3x^2 y - \frac{1}{2} xy^2 \right]_0^2 dx \\ &= \int_0^1 (6x^2 - 2x) dx \\ &= [2x^3 - x^2]_0^1 \\ &= 1 \end{aligned}$$

(7)
$$\begin{aligned} \iint_D (3xy^2 + 2x^3 y) dx dy &= \int_1^2 \left\{ \int_0^1 (3xy^2 + 2x^3 y) dy \right\} dx \\ &= \int_1^2 [xy^3 + x^3 y^2]_0^1 dx \\ &= \int_1^2 (x + x^3) dx \\ &= \left[\frac{1}{2} x^2 + \frac{1}{4} x^4 \right]_1^2 \\ &= \{2 + 4\} - \left\{ \frac{1}{2} + \frac{1}{4} \right\} \\ &= \frac{21}{4} \end{aligned}$$