第2章 3 「高次偏導関数」 第2回

解答

1. (1) $z_{xx} = -20x^3 + 6xy^3, z_{xy} = 9x^2y^2 + 4y$

(2) $z_{xx} = -\dfrac{2y}{(x+y)^3}, z_{xy} = \dfrac{x-y}{(x+y)^3}$

(3) $z_{xx} = -\sin(x+2y), z_{xy} = -2\sin(x+2y)$

(4) $z_{xx} = -\dfrac{\cos 3y}{x^2}, z_{xy} = -\dfrac{3\sin 3y}{x}$

2. (1) $z_{xx} = 6x, z_{xy} = -4y, z_{yx} = -4y,$

$z_{yy} = -4x + 12y^2$

(2) $z_{xx} = -4e^{3y}\sin 2x, z_{xy} = 6e^{3y}\cos 2x$

$z_{yx} = 6e^{3y}\cos 2x, z_{yy} = 9e^{3y}\sin 2x$

解説

1. (1) $z_x = (-x^5 + x^3y^3 + 2xy^2)_x$

$= -5x^4 + 3x^2y^3 + 2y^2$

$z_{xx} = (-5x^4 + 3x^2y^3 + 2y^2)_x = -20x^3 + 6xy^3$

$z_{xy} = (-5x^4 + 3x^2y^3 + 2y^2)_y = 9x^2y^2 + 4y$

(2) $z_x = \dfrac{(x)_x(x+y) - x(x+y)_x}{(x+y)^2}$

$= \dfrac{x+y-x}{(x+y)^2} = \dfrac{y}{(x+y)^2}$

$z_{xx} = \{y(x+y)^{-2}\}_x$

$= (y)_x(x+y)^{-2} + y\{(x+y)^{-2}\}_x$

$= -2y(x+y)^{-3}(x+y)_x = -\dfrac{2y}{(x+y)^3}$

$z_{xy} = \{y(x+y)^{-2}\}_y$

$= (y)_y(x+y)^{-2} + y\{(x+y)^{-2}\}_y$

$= (x+y)^{-2} - 2y(x+y)^{-3}(x+y)_y$

$= \dfrac{1}{(x+y)^2} - \dfrac{2y}{(x+y)^3} = \dfrac{x+y-2y}{(x+y)^3}$

$= \dfrac{x-y}{(x+y)^3}$

または, 商の微分法を用いて

$z_{xy} = \dfrac{(y)_y(x+y)^2 - y\{(x+y)^2\}_y}{\{(x+y)^2\}^2}$

$= \dfrac{(x+y)^2 - 2y(x+y)}{(x+y)^4} = \dfrac{(x+y)-2y}{(x+y)^3}$

$= \dfrac{x-y}{(x+y)^3}$ としてもよい. z_{xx} も同様.

(3) $z_x = \{\cos(x+2y)\}(x+2y)_x = \cos(x+2y)$

$z_{xx} = \{\cos(x+2y)\}_x$

$= -\{\sin(x+2y)\}(x+2y)_x = -\sin(x+2y)$

$z_{xy} = \{\cos(x+2y)\}_y$

$= -\{\sin(x+2y)\}(x+2y)_y = -2\sin(x+2y)$

(4) $z_x = (\log x)_x \cos 3y = \dfrac{1}{x}\cos 3y = \dfrac{\cos 3y}{x}$

$z_{xx} = \left(\dfrac{1}{x}\cos 3y\right)_x = (x^{-1})_x \cos 3y$

$= -x^{-2}\cos 3y = -\dfrac{1}{x^2}\cos 3y = -\dfrac{\cos 3y}{x^2}$

$z_{xy} = \left(\dfrac{1}{x}\cos 3y\right)_y = \dfrac{1}{x}(\cos 3y)_y$

$= \dfrac{1}{x}(-\sin 3y)(3y)_y = -\dfrac{3}{x}\sin 3y = -\dfrac{3\sin 3y}{x}$

2. (1) $z_x = (x^3 - 2xy^2 + y^4)_x = 3x^2 - 2y^2$

$z_{xx} = (3x^2 - 2y^2)_x = 6x$

$z_{xy} = (3x^2 - 2y^2)_y = -4y$

$z_y = (x^3 - 2xy^2 + y^4)_y = -4xy + 4y^3$

$z_{yx} = (-4xy + 4y^3)_x = -4y$

$z_{yy} = (-4xy + 4y^3)_y = -4x + 12y^2$

(2) $z_x = (e^{3y}\sin 2x)_x = 2e^{3y}\cos 2x$

$z_{xx} = (2e^{3y}\cos 2x)_x = -4e^{3y}\sin 2x$

$z_{xy} = (2e^{3y}\cos 2x)_y = 6e^{3y}\cos 2x$

$z_y = (e^{3y}\sin 2x)_y = 3e^{3y}\sin 2x$

$z_{yx} = (3e^{3y}\sin 2x)_x = 6e^{3y}\cos 2x$

$z_{yy} = (3e^{3y}\sin 2x)_y = 9e^{3y}\sin 2x$