第3章 4 「部分積分法」 第3回

解答

1. (1) e^2+1 (2) π

(3) $\dfrac{1}{4}(e^2+1)$ (4) $2\pi-4$

2. (1) $\dfrac{1}{3}(2e^3+1)$ (2) $\dfrac{1}{4}(3e^4+1)$

解説

1. (1) $\displaystyle\int_0^2 xe^x dx$

$$
\begin{aligned}
&= \Big[xe^x\Big]_0^2 - \int_0^2 (x)'e^x dx \\
&= \Big[xe^x\Big]_0^2 - \int_0^2 e^x dx \\
&= \Big[xe^x\Big]_0^2 - \Big[e^x\Big]_0^2 \\
&= (2e^2-0)-(e^2-1) \\
&= e^2+1
\end{aligned}
$$

(2) $\displaystyle\int_0^\pi x\sin x dx$

$$
\begin{aligned}
&= \Big[x(-\cos x)\Big]_0^\pi - \int_0^\pi (x)'(-\cos x)dx \\
&= \Big[-x\cos x\Big]_0^\pi + \int_0^\pi \cos x dx \\
&= \Big[-x\cos x\Big]_0^\pi + \Big[\sin x\Big]_0^\pi \\
&= (\pi-0)+(0-0) \\
&= \pi
\end{aligned}
$$

(3) $\displaystyle\int_0^1 xe^{2x} dx$

$$
\begin{aligned}
&= \left[x\left(\frac{1}{2}e^{2x}\right)\right]_0^1 - \int_0^1 (x)'\frac{1}{2}e^{2x} dx \\
&= \left[\frac{1}{2}xe^{2x}\right]_0^1 - \int_0^1 \frac{1}{2}e^{2x} dx \\
&= \left[\frac{1}{2}xe^{2x}\right]_0^1 - \left[\frac{1}{4}e^{2x}\right]_0^1 \\
&= \left(\frac{1}{2}e^2-0\right)-\left(\frac{1}{4}e^2-\frac{1}{4}\right) \\
&= \frac{1}{4}(e^2+1)
\end{aligned}
$$

(4) $\displaystyle\int_0^\pi x\cos\frac{x}{2} dx$

$$
\begin{aligned}
&= \left[x\left(2\sin\frac{x}{2}\right)\right]_0^\pi - \int_0^\pi (x)'\left(2\sin\frac{x}{2}\right) dx \\
&= \left[2x\sin\frac{x}{2}\right]_0^\pi - \int_0^\pi 2\sin\frac{x}{2} dx \\
&= \left[2x\sin\frac{x}{2}\right]_0^\pi + \left[4\cos\frac{x}{2}\right]_0^\pi \\
&= (2\pi-0)+(0-4) \\
&= 2\pi-4
\end{aligned}
$$

2. (1) $\displaystyle\int_1^e 3x^2\log x dx$

$$
\begin{aligned}
&= \Big[x^3\log x\Big]_1^e - \int_1^e x^3(\log x)' dx \\
&= \Big[x^3\log x\Big]_1^e - \int_1^e x^2 dx \\
&= \Big[x^3\log x\Big]_1^e - \left[\frac{1}{3}x^3\right]_1^e \\
&= (e^3-0)-\left(\frac{1}{3}e^3-\frac{1}{3}\right) \\
&= \frac{1}{3}(2e^3+1)
\end{aligned}
$$

(2) $\displaystyle\int_1^e 4x^3\log x dx$

$$
\begin{aligned}
&= \Big[x^4\log x\Big]_1^e - \int_1^e x^4(\log x)' dx \\
&= \Big[x^4\log x\Big]_1^e - \int_1^e x^3 dx \\
&= \Big[x^4\log x\Big]_1^e - \left[\frac{1}{4}x^4\right]_1^e \\
&= (e^4-0)-\left(\frac{1}{4}e^4-\frac{1}{4}\right) \\
&= \frac{1}{4}(3e^4+1)
\end{aligned}
$$