第1章 7 「逆三角関数とその導関数」 第3回

解答

1. (1) 0 (2) $-\dfrac{\pi}{3}$

(3) $\dfrac{\pi}{4}$ (4) $\dfrac{2}{3}\pi$

(5) $\dfrac{\pi}{6}$ (6) $-\dfrac{\pi}{3}$

2. (1) $y' = \dfrac{4}{\sqrt{1-16x^2}}$ (2) $y' = \dfrac{2x}{\sqrt{1-x^4}}$

(3) $y' = -\dfrac{\sqrt{2}}{\sqrt{1-2x^2}}$ (4) $y' = -\dfrac{1}{2\sqrt{x-x^2}}$

(5) $y' = \dfrac{4}{1+16x^2}$ (6) $y' = \dfrac{2}{4+x^2}$

解説

1. 逆三角関数の定義式

$$y = \sin^{-1} x \quad \Leftrightarrow \quad \sin y = x \quad \left(-\frac{\pi}{2} \leqq y \leqq \frac{\pi}{2}\right)$$

$$y = \cos^{-1} x \quad \Leftrightarrow \quad \cos y = x \quad (0 \leqq y \leqq \pi)$$

$$y = \tan^{-1} x \quad \Leftrightarrow \quad \tan y = x \quad \left(-\frac{\pi}{2} < y < \frac{\pi}{2}\right)$$

を利用して，値を求めればよい．

(1) $y = \sin^{-1} 0 \quad \Leftrightarrow \quad \sin y = 0$

これから $y = 0$ となる．

よって $\sin^{-1} 0 = 0$

(2) $y = \sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right) \quad \Leftrightarrow \quad \sin y = -\dfrac{\sqrt{3}}{2}$

これから $y = -\dfrac{\pi}{3}$ となる．

よって $\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right) = -\dfrac{\pi}{3}$

(3) $y = \cos^{-1} \dfrac{1}{\sqrt{2}} \quad \Leftrightarrow \quad \cos y = \dfrac{1}{\sqrt{2}}$

これから $y = \dfrac{\pi}{4}$ となる．

よって $\cos^{-1} \dfrac{1}{\sqrt{2}} = \dfrac{\pi}{4}$

(4) $y = \cos^{-1}\left(-\dfrac{1}{2}\right) \quad \Leftrightarrow \quad \cos y = -\dfrac{1}{2}$

これから $y = \dfrac{2}{3}\pi$ となる．

よって $\cos^{-1}\left(-\dfrac{1}{2}\right) = \dfrac{2}{3}\pi$

(5) $y = \tan^{-1} \dfrac{1}{\sqrt{3}} \quad \Leftrightarrow \quad \tan y = \dfrac{1}{\sqrt{3}}$

これから $y = \dfrac{\pi}{6}$ となる．

よって $\tan^{-1} \dfrac{1}{\sqrt{3}} = \dfrac{\pi}{6}$

(6) $y = \tan^{-1}(-\sqrt{3}) \quad \Leftrightarrow \quad \tan y = -\sqrt{3}$

これから $y = -\dfrac{\pi}{3}$ となる．

よって $\tan^{-1}(-\sqrt{3}) = -\dfrac{\pi}{3}$

2. (1) $y = \sin^{-1} u, \quad u = 4x$

$$\frac{dy}{du} = \frac{1}{\sqrt{1-u^2}}, \quad \frac{du}{dx} = 4 \quad \text{より}$$

$$y' = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{4}{\sqrt{1-u^2}}$$

$$= \frac{4}{\sqrt{1-(4x)^2}} = \frac{4}{\sqrt{1-16x^2}}$$

(2) $y = \sin^{-1} u, \quad u = x^2$

$$\frac{dy}{du} = \frac{1}{\sqrt{1-u^2}}, \quad \frac{du}{dx} = 2x \quad \text{より}$$

$$y' = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{2x}{\sqrt{1-u^2}}$$

$$= \frac{2x}{\sqrt{1-(x^2)^2}} = \frac{2x}{\sqrt{1-x^4}}$$

(3) $y = \cos^{-1} u, \quad u = \sqrt{2}x$

$$\frac{dy}{du} = -\frac{1}{\sqrt{1-u^2}}, \quad \frac{du}{dx} = \sqrt{2} \quad \text{より}$$

$$y' = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = -\frac{\sqrt{2}}{\sqrt{1-u^2}}$$

$$= -\frac{\sqrt{2}}{\sqrt{1-(\sqrt{2}x)^2}} = -\frac{\sqrt{2}}{\sqrt{1-2x^2}}$$

(4) $y = \cos^{-1} u, \quad u = \sqrt{x}$

$$\frac{dy}{du} = -\frac{1}{\sqrt{1-u^2}}, \quad \frac{du}{dx} = \frac{1}{2\sqrt{x}} \quad \text{より}$$

$$y' = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = -\frac{1}{\sqrt{1-u^2}} \cdot \frac{1}{2\sqrt{x}}$$

$$= -\frac{1}{2\sqrt{x}\sqrt{1-(\sqrt{x})^2}} = -\frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$= -\frac{1}{2\sqrt{x-x^2}}$$

(5) $y = \tan^{-1} u, \quad u = 4x$

$$\frac{dy}{du} = \frac{1}{1+u^2}, \quad \frac{du}{dx} = 4 \quad \text{より}$$

$$y' = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{4}{1+u^2}$$

$$= \frac{4}{1+(4x)^2} = \frac{4}{1+16x^2}$$

(6) $y = \tan^{-1} u, \quad u = \dfrac{x}{2}$

$$\frac{dy}{du} = \frac{1}{1+u^2}, \quad \frac{du}{dx} = \frac{1}{2} \quad \text{より}$$

$$y' = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{1+u^2} \cdot \frac{1}{2}$$

$$= \frac{1}{2}\frac{1}{1+\left(\dfrac{x}{2}\right)^2} = \frac{2}{4+x^2}$$