第１章 5 「合成関数の導関数」 第２回

解答

1. (1) $y = u^5, \quad u = x^2 + x + 1$

(2) $y = \cos u, \quad u = \log x$

2. (1) $y' = 16(4x+3)^3$ (2) $y' = 12x(2x^2+3)^2$

(3) $y' = 3(x^2+x+1)^2(2x+1)$

(4) $y' = 8x^3(x^4-1)$

(5) $y' = 2xe^{x^2+1}$ (6) $y' = 2e^{2\sin x}\cos x$

(7) $y' = \dfrac{2x}{x^2+4}$ (8) $y' = \dfrac{2x+1}{x^2+x+1}$

(9) $y' = -\tan x$

(10) $y' = \dfrac{2x+1}{2\sqrt{x^2+x+1}}$

(11) $y' = -\dfrac{x}{\sqrt{(x^2+3)^3}}$

(12) $y' = 2x\cos(x^2+2)$

(13) $y' = 2\sin x\cos x$ (14) $y' = -5\cos^4 x\sin x$

解説

1. (1) $y = (x^2+x+1)^5$ において

$u = x^2+x+1$ とおくと $y = u^5$ となる．

(2) $y = \cos(\log x)$ において

$u = \log x$ とおくと $y = \cos u$ となる．

2. (1) $y = u^4, \quad u = 4x+3$

$\dfrac{dy}{du} = 4u^3, \quad \dfrac{du}{dx} = 4$ より

$y' = \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = 4u^3 \cdot 4 = 16(4x+3)^3$

(2) $y = u^3, \quad u = 2x^2+3$

$\dfrac{dy}{du} = 3u^2, \quad \dfrac{du}{dx} = 4x$ より

$y' = \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

$= 3u^2 \cdot (4x) = 12x(2x^2+3)^2$

(3) $y = u^3, \quad u = x^2+x+1$

$\dfrac{dy}{du} = 3u^2, \quad \dfrac{du}{dx} = 2x+1$ より

$y' = \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

$= 3u^2 \cdot (2x+1) = 3(x^2+x+1)^2(2x+1)$

(4) $y = u^2, \quad u = x^4-1$

$\dfrac{dy}{du} = 2u, \quad \dfrac{du}{dx} = 4x^3$ より

$y' = \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = 2u\cdot(4x^3) = 8x^3(x^4-1)$

(5) $y = e^u, \quad u = x^2+1$

$\dfrac{dy}{du} = e^u, \quad \dfrac{du}{dx} = 2x$ より

$y' = \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = e^u \cdot (2x) = 2xe^{x^2+1}$

(6) $y = e^u, \quad u = 2\sin x$

$\dfrac{dy}{du} = e^u, \quad \dfrac{du}{dx} = 2\cos x$ より

$y' = \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = e^u\cdot(2\cos x) = 2e^{2\sin x}\cos x$

(7) $y = \log u, \quad u = x^2+4$

$\dfrac{dy}{du} = \dfrac{1}{u}, \quad \dfrac{du}{dx} = 2x$ より

$y' = \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = \dfrac{1}{u}\cdot(2x) = \dfrac{2x}{x^2+4}$

(8) $y = \log u, \quad u = x^2+x+1$

$\dfrac{dy}{du} = \dfrac{1}{u}, \quad \dfrac{du}{dx} = 2x+1$ より

$y' = \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = \dfrac{1}{u}\cdot(2x+1) = \dfrac{2x+1}{x^2+x+1}$

(9) $y = \log|u|, \quad u = \cos x$

$\dfrac{dy}{du} = \dfrac{1}{u}, \quad \dfrac{du}{dx} = -\sin x$ より

$y' = \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

$= \dfrac{1}{u}\cdot(-\sin x) = -\dfrac{\sin x}{\cos x} = -\tan x$

(10) $y = \sqrt{u}, \quad u = x^2+x+1$

$\dfrac{dy}{du} = \dfrac{1}{2\sqrt{u}}, \quad \dfrac{du}{dx} = 2x+1$ より

$y' = \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

$= \dfrac{1}{2\sqrt{u}}\cdot(2x+1) = \dfrac{2x+1}{2\sqrt{x^2+x+1}}$

(11) $y = \dfrac{1}{\sqrt{u}}, \quad u = x^2+3$

$\dfrac{dy}{du} = -\dfrac{1}{2\sqrt{u^3}}, \quad \dfrac{du}{dx} = 2x$ より

$y' = \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

$= -\dfrac{1}{2\sqrt{u^3}}\cdot(2x) = -\dfrac{x}{\sqrt{(x^2+3)^3}}$

(12) $y = \sin u, \quad u = x^2+2$

$\dfrac{dy}{du} = \cos u, \quad \dfrac{du}{dx} = 2x$ より

$y' = \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

$= \cos u\cdot(2x) = 2x\cos(x^2+2)$

(13) $y = u^2, \quad u = \sin x$

$\dfrac{dy}{du} = 2u, \quad \dfrac{du}{dx} = \cos x$ より

$y' = \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

$= 2u\cdot(\cos x) = 2\sin x\cos x$

(14) $y = u^5, \quad u = \cos x$

$\dfrac{dy}{du} = 5u^4, \quad \dfrac{du}{dx} = -\sin x$ より

$y' = \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

$= 5u^4\cdot(-\sin x) = -5\cos^4 x\sin x$