第１章 4 「三角関数の導関数」「指数関数と対数関数の導関数」 第３回

解答

1. (1) $-\sin x - \dfrac{1}{\cos^2 x}$　(2) $\sin x + x\cos x$

(3) $4\sin(1-4x)$　(4) $\dfrac{3}{\cos^2(3x+1)}$

(5) $-4e^{-4x}$　(6) $2xe^{2x}(1+x)$

(7) $-\dfrac{1}{4\sqrt[4]{e^x}}$　(8) $\dfrac{e^x(x-1)}{x^2}$

2. (1) 5　(2) -3

(3) $-\dfrac{2}{3}$

3. (1) $\log x + 1 + \dfrac{1}{x}$　(2) $\dfrac{6}{3x+2}$

(3) $\dfrac{1}{x-2}$　(4) $\dfrac{4}{4x-3}$

(5) $8^x \log 8$　(6) $-\left(\dfrac{1}{5}\right)^x \log 5$

(7) $\dfrac{1}{x\log 6}$　(8) $\dfrac{4}{(4x+1)\log 3}$

(9) $\dfrac{2}{2x-5}$　(10) $\dfrac{1}{x-4}$

解説

1. (1) $y' = (\cos x)' - (\tan x)' = -\sin x - \dfrac{1}{\cos^2 x}$

(2) $y' = (x)'\sin x + x(\sin x)' = \sin x + x\cos x$

(3) $y' = (-4)\cdot\{-\sin(1-4x)\} = 4\sin(1-4x)$

(4) $y' = 3\cdot\dfrac{1}{\cos^2(3x+1)} = \dfrac{3}{\cos^2(3x+1)}$

(5) $y' = (-4)\cdot e^{-4x} = -4e^{-4x}\ \left(= -\dfrac{4}{e^{4x}}\right)$

(6) $y' = (x^2)'e^{2x} + x^2(e^{2x})' = 2xe^{2x} + 2x^2e^{2x}$
$= 2xe^{2x}(1+x)$

(7) $y' = (e^{-\frac{1}{4}x})' = -\dfrac{1}{4}\cdot e^{-\frac{1}{4}x} = -\dfrac{1}{4\sqrt[4]{e^x}}$

(8) $y' = \dfrac{(e^x)'x - e^x(x)'}{x^2} = \dfrac{xe^x - e^x}{x^2}$
$= \dfrac{e^x(x-1)}{x^2}$

2. (1) $\log e^5 = 5\log e = 5$

(2) $\log\dfrac{1}{e^3} = \log e^{-3} = -3\log e = -3$

(3) $\log\dfrac{1}{\sqrt[3]{e^2}} = \log e^{-\frac{2}{3}} = -\dfrac{2}{3}\log e = -\dfrac{2}{3}$

3. (1) $y' = (x+1)'\log x + (x+1)\cdot(\log x)'$
$= 1\cdot\log x + (x+1)\cdot\dfrac{1}{x} = \log x + \dfrac{x+1}{x}$
$= \log x + 1 + \dfrac{1}{x}$

(2) $y' = 3\cdot\dfrac{2}{3x+2} = \dfrac{6}{3x+2}$

(3) $y' = (-1)\cdot\dfrac{1}{-x+2} = -\dfrac{1}{-x+2} = \dfrac{1}{x-2}$

(4) $y' = -4\cdot\dfrac{1}{-4x+3} = \dfrac{4}{4x-3}$

(5) $(a^x) = a^x\log a$ を用いて $y' = 8^x\log 8$

(6) $y = 5^{-x}$ より $y' = -1\cdot 5^{-x}\log 5$
$= -5^{-x}\log 5 = -\left(\dfrac{1}{5}\right)^x\log 5$

(7) $(\log_a x)' = \dfrac{1}{x\log a}$ を用いて $y' = \dfrac{1}{x\log 6}$

(8) $y' = 4\cdot\dfrac{1}{(4x+1)\log 3} = \dfrac{4}{(4x+1)\log 3}$

(9) $y' = 2\cdot\dfrac{1}{2x-5} = \dfrac{2}{2x-5}$

(10) $y' = -1\cdot\dfrac{1}{-x+4} = \dfrac{1}{x-4}$