第１章 3 「導関数の性質」 第２回

解答

1. (1) $y' = 8x$ (2) $y' = 3x^2$

(3) $y' = 2x - \dfrac{3}{2}$ (4) $y' = 4x + 5$

(5) $y' = 9x^2 + 4x + 3$ (6) $y' = \dfrac{3}{(x+1)^2}$

(7) $y' = -\dfrac{1}{(x+4)^2}$ (8) $y' = \dfrac{5}{(x+2)^2}$

2. (1) $-\dfrac{6}{x^4}$ (2) $-12x^{-4}$

(3) $\dfrac{1}{4}x^{-\frac{3}{4}}$ (4) $\dfrac{1}{3}x^{-\frac{2}{3}}$

(5) $\dfrac{2}{3\sqrt[3]{x}}$ (6) $\dfrac{7}{2}\sqrt{x^5}$

(7) $5(x+1)^4$ (8) $8(2x-1)^3$

(9) $6(4x+3)^{\frac{1}{2}}$ (10) $\dfrac{9}{2}\sqrt{3x+1}$

(11) $-\dfrac{9}{(3x-1)^4}$ (12) $-\dfrac{16}{(4x-1)^3}$

解説

1. (1) $y' = 4(x^2)' = 4 \cdot 2x = 8x$

(2) $y' = (x^3)' - (3)' = 3x^2 - 0 = 3x^2$

(3) $y' = \left(x^2 - \dfrac{3}{2}x\right)' = (x^2)' - \left(\dfrac{3}{2}x\right)' = 2x - \dfrac{3}{2}$

(4) $y' = (2x-1)'(x+3) + (2x-1)(x+3)'$
$= 2 \cdot (x+3) + (2x-1) \cdot 1 = 4x + 5$

(5) $y' = (3x+2)'(x^2+1) + (3x+2)(x^2+1)'$
$= 3 \cdot (x^2+1) + (3x+2) \cdot 2x = 9x^2 + 4x + 3$

(6) $y' = \dfrac{(3x)'(x+1) - 3x(x+1)'}{(x+1)^2}$
$= \dfrac{3 \cdot (x+1) - 3x \cdot 1}{(x+1)^2} = \dfrac{3}{(x+1)^2}$

(7) $y' = \dfrac{(1)'(x+4) - (x+4)'}{(x+4)^2} = -\dfrac{1}{(x+4)^2}$

または $y' = -\dfrac{(x+4)'}{(x+4)^2} = -\dfrac{1}{(x+4)^2}$

(8) $y' = \dfrac{(2x-1)'(x+2) - (2x-1)(x+2)'}{(x+2)^2}$
$= \dfrac{2(x+2) - (2x-1) \cdot 1}{(x+2)^2} = \dfrac{5}{(x+2)^2}$

2. (1) $y' = (2x^{-3})' = 2 \cdot (-3)x^{-4} = -6x^{-4} = -\dfrac{6}{x^4}$

(2) $y' = 4 \cdot (-3)x^{-4} = -12x^{-4} \left(= -\dfrac{12}{x^4}\right)$

(3) $y' = \dfrac{1}{4}x^{\frac{1}{4}-1} = \dfrac{1}{4}x^{-\frac{3}{4}} \left(= \dfrac{1}{4\sqrt[4]{x^3}}\right)$

(4) $y' = \dfrac{1}{3}x^{\frac{1}{3}-1} = \dfrac{1}{3}x^{-\frac{2}{3}} \left(= \dfrac{1}{3\sqrt[3]{x^2}}\right)$

(5) $y' = (x^{\frac{2}{3}})' = \dfrac{2}{3}x^{-\frac{1}{3}} = \dfrac{2}{3\sqrt[3]{x}}$

(6) $y' = \left(x^3 \cdot x^{\frac{1}{2}}\right)' = \left(x^{\frac{7}{2}}\right)' = \dfrac{7}{2}x^{\frac{7}{2}-1}$
$= \dfrac{7}{2}x^{\frac{5}{2}} = \dfrac{7}{2}\sqrt{x^5}$

(7) $y' = 1 \cdot 5(x+1)^4 = 5(x+1)^4$

(8) $y' = 2 \cdot 4(2x-1)^3 = 8(2x-1)^3$

(9) $y' = 4 \cdot \dfrac{3}{2}(4x+3)^{\frac{1}{2}} = 6(4x+3)^{\frac{1}{2}}$
$\left(= 6\sqrt{4x+3}\right)$

(10) $y' = \left\{(3x+1)^{\frac{3}{2}}\right\}' = 3 \cdot \dfrac{3}{2}(3x+1)^{\frac{1}{2}}$
$= \dfrac{9}{2}\sqrt{3x+1}$

(11) $y' = \left\{(3x-1)^{-3}\right\}' = 3 \cdot (-3) \cdot (3x-1)^{-4}$
$= -9(3x-1)^{-4} = -\dfrac{9}{(3x-1)^4}$

(12) $y' = \left\{2(4x-1)^{-2}\right\}' = 2 \cdot 4 \cdot (-2) \cdot (4x-1)^{-3}$
$= -16(4x-1)^{-3} = -\dfrac{16}{(4x-1)^3}$