第3章 3 「行列の積の行列式」 第2回

解答

1. (1) 6 (2) 9 (3) 4
(4) 18 (5) 72 (6) 36

2. (1) 24 (2) 24 (3) 96

3. (1) -2 (2) 5 (3) -10
(4) -10 (5) -10 (6) -50

4. (1) $\frac{1}{4}$ (2) $-\frac{1}{3}$ (3) $-\frac{4}{3}$
(4) $-\frac{1}{12}$ (5) -3 (6) 4

解説

1. (1) $|AB| = |A||B| = 3 \times 2 = 6$
(2) $|A^2| = |AA| = |A||A| = |A|^2 = 3^2 = 9$
(3) $|B^2| = |BB| = |B||B| = |B|^2 = 2^2 = 4$
(4) $|ABA| = |A||B||A| = 3 \times 2 \times 3 = 18$
(5) $|AB^3A| = |A||B|^3|A| = 3 \times 2^3 \times 3 = 72$
(6) $|BABA| = |B||A||B||A| = 2 \times 3 \times 2 \times 3 = 36$

2. (1) $|ABC| = |A||B||C| = 2 \times 4 \times 3 = 24$
(2) $|CAB| = |C||A||B| = 3 \times 2 \times 4 = 24$
(3) $|AB^2C| = |A||B|^2|C| = 2 \times 4^2 \times 3 = 96$

3. (1) $|{}^tA| = |A| = -2$
(2) $|{}^tB| = |B| = 5$
(3) $|A{}^tB| = |A||{}^tB| = |A||B| = -2 \times 5 = -10$
(4) $|B{}^tA| = |B||{}^tA| = |B||A| = 5 \times (-2) = -10$
(5) $|{}^tA{}^tB| = |{}^tA||{}^tB| = |A||B| = -2 \times 5 = -10$
(6) $|{}^tAB{}^tB| = |{}^tA||B||{}^tB| = |A||B||B|$
$= -2 \times 5 \times 5 = -50$

4. (1) $|A^{-1}| = \frac{1}{|A|} = \frac{1}{4}$
(2) $|B^{-1}| = \frac{1}{|B|} = -\frac{1}{3}$
(3) $|AB^{-1}| = |A||B^{-1}| = |A|\frac{1}{|B|} = -\frac{4}{3}$
(4) $|A^{-1}B^{-1}| = |A^{-1}||B^{-1}| = \frac{1}{|A|}\frac{1}{|B|} = -\frac{1}{12}$
(5) $|ABA^{-1}| = |A||B||A^{-1}| = |A||B|\frac{1}{|A|}$
$= |B| = -3$
(6) $|B^{-1}AB| = |B^{-1}||A||B| = \frac{1}{|B|}|A||B| = |A| = 4$