第 3 章 4 「部分積分法」 第 2 回

解答

1. (1) $\dfrac{2}{e}$　(2) -2

(3) $e^2+\dfrac{2}{e}$　(4) 2

2. (1) $\dfrac{1}{2}(3e^4+1)$　(2) e^2

解説

1. (1) $\displaystyle\int_{-1}^{1} xe^x dx$

$$
\begin{aligned}
&= \Big[xe^x\Big]_{-1}^{1} - \int_{-1}^{1}(x)'e^x dx \\
&= \Big[xe^x\Big]_{-1}^{1} - \int_{-1}^{1} e^x dx \\
&= \Big[xe^x\Big]_{-1}^{1} - \Big[e^x\Big]_{-1}^{1} \\
&= \left(e+e^{-1}\right) - \left(e-e^{-1}\right) \\
&= 2e^{-1} = \frac{2}{e}
\end{aligned}
$$

(2) $\displaystyle\int_{0}^{\pi} x\cos x dx$

$$
\begin{aligned}
&= \Big[x\sin x\Big]_{0}^{\pi} - \int_{0}^{\pi}(x)'\sin x dx \\
&= \Big[x\sin x\Big]_{0}^{\pi} - \int_{0}^{\pi}\sin x dx \\
&= \Big[x\sin x\Big]_{0}^{\pi} - \Big[-\cos x\Big]_{0}^{\pi} \\
&= (0-0)-(1+1) = -2
\end{aligned}
$$

(3) $\displaystyle\int_{-1}^{2} xe^x dx$

$$
\begin{aligned}
&= \Big[xe^x\Big]_{-1}^{2} - \int_{-1}^{2}(x)'e^x dx \\
&= \Big[xe^x\Big]_{-1}^{2} - \int_{-1}^{2} e^x dx \\
&= \Big[xe^x\Big]_{-1}^{2} - \Big[e^x\Big]_{-1}^{2} \\
&= \left(2e^2+e^{-1}\right) - \left(e^2-e^{-1}\right) \\
&= e^2+2e^{-1} = e^2+\frac{2}{e}
\end{aligned}
$$

(4) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\sin x dx$

$$
\begin{aligned}
&= \Big[x(-\cos x)\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(x)'(-\cos x)dx \\
&= \Big[-x\cos x\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos x dx \\
&= \Big[-x\cos x\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \Big[\sin x\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= (0-0)+(1+1) \\
&= 2
\end{aligned}
$$

2. (1) $\displaystyle\int_{1}^{e^2} 2x\log x dx$

$$
\begin{aligned}
&= \Big[x^2\log x\Big]_{1}^{e^2} - \int_{1}^{e^2} x^2(\log x)' dx \\
&= \Big[x^2\log x\Big]_{1}^{e^2} - \int_{1}^{e^2} x dx \\
&= \Big[x^2\log x\Big]_{1}^{e^2} - \Big[\frac{1}{2}x^2\Big]_{1}^{e^2} \\
&= \left(2e^4-0\right) - \left(\frac{1}{2}e^4-\frac{1}{2}\right) \\
&= \frac{1}{2}(3e^4+1)
\end{aligned}
$$

(2) $\displaystyle\int_{e}^{e^2} \log x dx$

$$
\begin{aligned}
&= \Big[x\log x\Big]_{e}^{e^2} - \int_{e}^{e^2} x(\log x)' dx \\
&= \Big[x\log x\Big]_{e}^{e^2} - \int_{e}^{e^2} 1 dx \\
&= \Big[x\log x\Big]_{e}^{e^2} - \Big[x\Big]_{e}^{e^2} \\
&= \left(2e^2-e\right) - \left(e^2-e\right) \\
&= e^2
\end{aligned}
$$