第3章 3 「置換積分法」 第2回

解答

1. (1) $-\dfrac{1}{7}\cos^7 x + C$

(2) $\dfrac{1}{24}(3x-2)^8 + C$

(3) $\dfrac{2}{3}\sqrt{(x^2-1)^3} + C$

(4) $-\dfrac{1}{4}e^{-2x^2} + C$

(5) $\log(2-\cos x) + C$

(6) $\log|x^3+x^2+x+1| + C$

2. (1) $\dfrac{1}{7}$　(2) $\dfrac{1}{16}$

(3) $\dfrac{2}{3}(\sqrt{2}-1)$　(4) $\log 2$

解説

1. (1) $\cos x = t$ とおくと，

$-\sin x dx = dt$ より $\sin x dx = -dt$

$\displaystyle\int \cos^6 x \sin x dx = -\int t^6 dt$

$= -\dfrac{1}{7}t^7 + C = -\dfrac{1}{7}\cos^7 x + C$

(2) $3x-2 = t$ とおくと，

$3dx = dt$ より $dx = \dfrac{1}{3}dt$

$\displaystyle\int (3x-2)^7 dx = \frac{1}{3}\int t^7 dt$

$= \dfrac{1}{24}t^8 + C = \dfrac{1}{24}(3x-2)^8 + C$

(3) $x^2-1 = t$ とおくと，

$2xdx = dt$

$\displaystyle\int 2x\sqrt{x^2-1}dx = \int \sqrt{t}dt$

$= \dfrac{2}{3}t^{\frac{3}{2}} + C = \dfrac{2}{3}(x^2-1)^{\frac{3}{2}} + C$

$= \dfrac{2}{3}\sqrt{(x^2-1)^3} + C$

(4) $-2x^2 = t$ とおくと，

$-4xdx = dt$ より $xdx = -\dfrac{1}{4}dt$

$\displaystyle\int xe^{-2x^2}dx = -\frac{1}{4}\int e^t dt$

$= -\dfrac{1}{4}e^t + C = -\dfrac{1}{4}e^{-2x^2} + C$

(5) $2-\cos x = t$ とおくと，

$\sin x dx = dt$

$\displaystyle\int \frac{\sin x}{2-\cos x}dx = \int \frac{1}{t}dt$

$= \log|t| + C = \log|2-\cos x| + C$

$= \log(2-\cos x) + C$

(6) $x^3+x^2+x+1 = t$ とおくと，

$(3x^2+2x+1)dx = dt$

$\displaystyle\int \frac{3x^2+2x+1}{x^3+x^2+x+1}dx = \int \frac{1}{t}dt$

$= \log|t| + C = \log|x^3+x^2+x+1| + C$

2. (1) $2x-1 = t$ とおくと，

$2dx = dt$ より $dx = \dfrac{1}{2}dt$

$\displaystyle\int_0^1 (2x-1)^6 dx = \frac{1}{2}\int_{-1}^1 t^6 dt = \frac{1}{7}$

(2) $\sin x = t$ とおくと，

$\cos x dx = dt$

$\displaystyle\int_0^{\frac{\pi}{4}} \sin^3 x \cos x dx = \int_0^{\frac{1}{\sqrt{2}}} t^3 dt = \frac{1}{16}$

(3) $x^3+1 = t$ とおくと，

$3x^2 dx = dt$ より $x^2 dx = \dfrac{1}{3}dt$

$\displaystyle\int_0^1 \frac{x^2}{\sqrt{x^3+1}}dx = \frac{1}{3}\int_1^2 \frac{1}{\sqrt{t}}dt = \frac{2}{3}(\sqrt{2}-1)$

(4) $2x^2-3x+4 = t$ とおくと，

$(4x-3)dx = dt$

$\displaystyle\int_1^2 \frac{4x-3}{2x^2-3x+4}dx = \int_3^6 \frac{1}{t}dt = \Big[\log|t|\Big]_3^6$

$= \log 6 - \log 3 = \log\dfrac{6}{3} = \log 2$