第3章 2 「定積分の計算」 第2回

解答

1. (1) 12　(2) 21

(3) $\dfrac{1}{6}$　(4) $\dfrac{10}{3}$

(5) $\dfrac{2}{3}$　(6) $\dfrac{1}{2}(e^2-e^{-2})$

(7) $\dfrac{1}{\sqrt{2}}$　(8) $\dfrac{1}{3}$

2. (1) 8　(2) 1

解説

1. (1) $\displaystyle\int_{-2}^{1}(-2x+3)dx$

$\displaystyle=\Big[-x^2+3x\Big]_{-2}^{1}=12$

(2) $\displaystyle\int_{-1}^{2}(9x^2-4x)dx$

$\displaystyle=\Big[3x^3-2x^2\Big]_{-1}^{2}=21$

(3) $\displaystyle\int_{0}^{1}(2x^2+3x-2)dx$

$\displaystyle=\Big[\frac{2}{3}x^3+\frac{3}{2}x^2-2x\Big]_{0}^{1}=\frac{1}{6}$

(4) $\displaystyle\int_{0}^{2}(x^3-x^2+x)dx$

$\displaystyle=\Big[\frac{1}{4}x^4-\frac{1}{3}x^3+\frac{1}{2}x^2\Big]_{0}^{2}=\frac{10}{3}$

(5) $\displaystyle\int_{1}^{3}\frac{1}{x^2}dx=\Big[-\frac{1}{x}\Big]_{1}^{3}=\frac{2}{3}$

(6) $\displaystyle\int_{-1}^{1}e^{-2x}dx=\Big[-\frac{1}{2}e^{-2x}\Big]_{-1}^{1}=\frac{1}{2}(e^2-e^{-2})$

(7) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}}\sin x dx=\Big[-\cos x\Big]_{-\frac{\pi}{4}}^{\frac{\pi}{2}}=\frac{1}{\sqrt{2}}$

(8) $\displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{6}}\cos 3x dx=\Big[\frac{1}{3}\sin 3x\Big]_{-\frac{\pi}{3}}^{\frac{\pi}{6}}=\frac{1}{3}$

2. (1) $x^4, x^2, 1$ は偶関数，x^3, x は奇関数だから

$\displaystyle\int_{-1}^{1}(5x^4-4x^3+6x^2-5x+1)dx$

$\displaystyle=2\int_{0}^{1}(5x^4+6x^2+1)dx=8$

(2) $\cos 2x$ は偶関数，$\sin 3x$ は奇関数だから

$\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}(\cos 2x-2\sin 3x)dx$

$\displaystyle=2\int_{0}^{\frac{\pi}{4}}\cos 2x dx=1$