

## 第3章 2 「定積分の計算」 第1回

解答

1. (1)  $\frac{5}{2}$                       (2)  $\frac{2}{3}$   
 (3) 1                              (4) -3  
 (5)  $\frac{52}{3}$                           (6)  $e - 1$   
 (7)  $\frac{1}{2}$                             (8)  $\frac{\sqrt{3}}{4}$   
 2. (1)  $\frac{2}{5}$                           (2) 2

(2)  $\cos x$  は偶関数,  $\sin x$  は奇関数だから

$$\begin{aligned} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos x + \sin x) dx &= 2 \int_0^{\frac{\pi}{2}} \cos x dx \\ &= 2 \left[ \sin x \right]_0^{\frac{\pi}{2}} = 2 \end{aligned}$$

解説

1. 教科書 p.94 例7のように考える.

- (1)  $\int_0^1 (x+2) dx = \left[ \frac{1}{2}x^2 + 2x \right]_0^1$   
 $= \left( \frac{1}{2} \cdot 1^2 + 2 \cdot 1 \right) - \left( \frac{1}{2} \cdot 0^2 + 2 \cdot 0 \right) = \frac{5}{2}$
- (2)  $\int_0^2 (x^2 - 1) dx = \left[ \frac{1}{3}x^3 - x \right]_0^2$   
 $= \left( \frac{1}{3} \cdot 2^3 - 2 \right) - \left( \frac{1}{3} \cdot 0^3 - 0 \right) = \frac{2}{3}$
- (3)  $\int_0^1 (3x^2 - 2x + 1) dx = \left[ x^3 - x^2 + x \right]_0^1$   
 $= (1^3 - 1^2 + 1) - (0^3 - 0^2 + 0) = 1$
- (4)  $\int_{-1}^2 (4x^3 - 6x^2) dx = \left[ x^4 - 2x^3 \right]_{-1}^2$   
 $= (2^4 - 2 \cdot 2^3) - \{ (-1)^4 - 2(-1)^3 \} = -3$
- (5)  $\int_1^9 \sqrt{x} dx = \left[ \frac{2}{3}x^{\frac{3}{2}} \right]_1^9$   
 $= \left( \frac{2}{3} \cdot 9^{\frac{3}{2}} \right) - \left( \frac{2}{3} \cdot 1^{\frac{3}{2}} \right) = \frac{2}{3} \cdot (3^2)^{\frac{3}{2}} - \frac{2}{3}$   
 $= \frac{2}{3} (3^3 - 1) = \frac{52}{3}$
- (6)  $\int_0^1 e^x dx = \left[ e^x \right]_0^1 = e^1 - e^0 = e - 1$
- (7)  $\int_0^{\frac{\pi}{3}} \sin x dx = \left[ -\cos x \right]_0^{\frac{\pi}{3}}$   
 $= -\cos \frac{\pi}{3} - (-\cos 0) = -\frac{1}{2} - (-1) = \frac{1}{2}$
- (8)  $\int_0^{\frac{\pi}{6}} \cos 2x dx = \left[ \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{6}}$   
 $= \frac{1}{2} \sin \frac{\pi}{3} - \frac{1}{2} \sin 0 = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot 0 = \frac{\sqrt{3}}{4}$

2. 教科書 p.95 例題4のように考える.

(1)  $x^4$  は偶関数,  $x^3$  は奇関数だから

$$\begin{aligned} \int_{-1}^1 (x^4 + x^3) dx &= 2 \int_0^1 x^4 dx \\ &= 2 \left[ \frac{1}{5}x^5 \right]_0^1 = \frac{2}{5} \end{aligned}$$