第１章 5 「合成関数の導関数」 第１回

解答

1. (1) $\dfrac{dy}{du}=4u^3,\quad \dfrac{du}{dx}=3,$
$y'=12(3x+1)^3$

(2) $\dfrac{dy}{du}=3u^2,\quad \dfrac{du}{dx}=2x+3,$
$y'=3(x^2+3x+1)^2(2x+3)$

(3) $\dfrac{dy}{du}=e^u,\quad \dfrac{du}{dx}=2,$
$y'=2e^{2x}$

(4) $\dfrac{dy}{du}=e^u,\quad \dfrac{du}{dx}=2x,$
$y'=2xe^{x^2}$

(5) $\dfrac{dy}{du}=e^u,\quad \dfrac{du}{dx}=-\sin x,$
$y'=-e^{\cos x}\sin x$

(6) $\dfrac{dy}{du}=\dfrac{1}{u},\quad \dfrac{du}{dx}=1,$
$y'=\dfrac{1}{x+1}$

(7) $\dfrac{dy}{du}=\dfrac{1}{u},\quad \dfrac{du}{dx}=2x,$
$y'=\dfrac{2x}{x^2+1}$

(8) $\dfrac{dy}{du}=\dfrac{1}{u},\quad \dfrac{du}{dx}=e^x,$
$y'=\dfrac{e^x}{e^x+1}$

(9) $\dfrac{dy}{du}=\dfrac{1}{2\sqrt{u}},\quad \dfrac{du}{dx}=2x,$
$y'=\dfrac{x}{\sqrt{x^2+1}}$

(10) $\dfrac{dy}{du}=-\dfrac{1}{2\sqrt{u^3}},\quad \dfrac{du}{dx}=2x,$
$y'=-\dfrac{x}{\sqrt{(x^2-1)^3}}$

(11) $\dfrac{dy}{du}=\cos u,\quad \dfrac{du}{dx}=2,$
$y'=2\cos 2x$

(12) $\dfrac{dy}{du}=3u^2,\quad \dfrac{du}{dx}=-\sin x,$
$y'=-3\cos^2 x\sin x$

解説

1. (1) $y'=\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$
$=4u^3\cdot 3=12(3x+1)^3$

(2) $y'=\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$
$=3u^2\cdot(2x+3)=3(x^2+3x+1)^2(2x+3)$

(3) $y'=\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$
$=e^u\cdot 2=2e^{2x}$

(4) $y'=\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$
$=e^u\cdot(2x)=2xe^{x^2}$

(5) $y'=\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$
$=e^u\cdot(-\sin x)=-e^{\cos x}\sin x$

(6) $y'=\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$
$=\dfrac{1}{u}\cdot 1=\dfrac{1}{x+1}$

(7) $y'=\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$
$=\dfrac{1}{u}\cdot(2x)=\dfrac{2x}{x^2+1}$

(8) $y'=\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$
$=\dfrac{1}{u}\cdot e^x=\dfrac{e^x}{e^x+1}$

(9) $y'=\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$
$=\dfrac{1}{2\sqrt{u}}\cdot(2x)=\dfrac{x}{\sqrt{x^2+1}}$

(10) $y'=\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$
$=-\dfrac{1}{2\sqrt{u^3}}\cdot(2x)=-\dfrac{x}{\sqrt{(x^2-1)^3}}$

(11) $y'=\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$
$=\cos u\cdot 2=2\cos 2x$

(12) $y'=\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$
$=3u^2\cdot(-\sin x)=-3\cos^2 x\sin x$