第１章 4 「三角関数の導関数」「指数関数と対数関数の導関数」 第２回

解答

1. (1) $\cos x + \dfrac{1}{\cos^2 x}$ (2) $2\cos(2x-1)$

(3) $-3\sin 3x$ (4) $\dfrac{4}{\cos^2(4x-1)}$

(5) $2e^{2x}$ (6) $-3e^{-3x}$

(7) $\dfrac{1}{2}\sqrt{e^x}$ (8) $-\dfrac{4}{e^{4x}}$

2. (1) 3 (2) -2

(3) $\dfrac{1}{2}$

3. (1) $3x^2\log x + x^2$ (2) $\dfrac{1}{x+1}$

(3) $\dfrac{2}{2x+3}$ (4) $\dfrac{3}{3x-1}$

(5) $6^x\log 6$ (6) $-\left(\dfrac{1}{4}\right)^x\log 4$

(7) $\dfrac{1}{x\log 4}$ (8) $\dfrac{2}{(2x-1)\log 2}$

(9) $\dfrac{1}{x+1}$ (10) $\dfrac{3}{3x-2}$

解説

1. (1) $y' = (\sin x)' + (\tan x)' = \cos x + \dfrac{1}{\cos^2 x}$

(2) $y' = 2\cdot\cos(2x-1) = 2\cos(2x-1)$

(3) $y' = 3\cdot(-\sin 3x) = -3\sin 3x$

(4) $y' = 4\cdot\dfrac{1}{\cos^2(4x-1)} = \dfrac{4}{\cos^2(4x-1)}$

(5) $y' = 2\cdot e^{2x} = 2e^{2x}$

(6) $y' = -3\cdot e^{-3x} = -3e^{-3x}\left(= -\dfrac{3}{e^{3x}}\right)$

(7) $y' = (e^{\frac{1}{2}x})' = \dfrac{1}{2}\cdot e^{\frac{1}{2}x} = \dfrac{1}{2}\sqrt{e^x}$

(8) $y' = (e^{-4x})' = -4\cdot e^{-4x} = -\dfrac{4}{e^{4x}}$

2. (1) $\log e^3 = 3\log e = 3$

(2) $\log\dfrac{1}{e^2} = \log e^{-2} = -2\log e = -2$

(3) $\log\sqrt{e} = \log e^{\frac{1}{2}} = \dfrac{1}{2}\log e = \dfrac{1}{2}$

3. (1) $y' = (x^3)'\log x + x^3(\log x)'$

$= 3x^2\log x + x^3\cdot\dfrac{1}{x} = 3x^2\log x + x^2$

(2) $y' = 1\cdot\dfrac{1}{x+1} = \dfrac{1}{x+1}$

(3) $y' = 2\cdot\dfrac{1}{2x+3} = \dfrac{2}{2x+3}$

(4) $y' = -3\cdot\dfrac{1}{-3x+1} = \dfrac{-3}{-3x+1} = \dfrac{3}{3x-1}$

(5) $(a^x) = a^x\log a$ を用いて $y' = 6^x\log 6$

(6) $y = 4^{-x}$ より $y' = -1\cdot 4^{-x}\log 4$

$= -4^{-x}\log 4 = -\left(\dfrac{1}{4}\right)^x\log 4$

(7) $(\log_a x)' = \dfrac{1}{x\log a}$ を用いて $y' = \dfrac{1}{x\log 4}$

(8) $y' = 2\cdot\dfrac{1}{(2x-1)\log 2} = \dfrac{2}{(2x-1)\log 2}$

(9) $(\log|x|)' = \dfrac{1}{x}$ を用いて

$y' = 1\cdot\dfrac{1}{x+1} = \dfrac{1}{x+1}$

(10) $y' = 3\cdot\dfrac{1}{3x-2} = \dfrac{3}{3x-2}$